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Consistent interactions of dual linearized gravity in $D = 5$: couplings with a topological BF model

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Abstract Under some plausible assumptions, we find that the dual formulation of linearized gravity in $D = 5$ can be nontrivially coupled to the topological BF model in such a way that the interacting theory exhibits a deformed gauge algebra and some deformed, on-shell reducibility relations. Moreover, the tensor field with the mixed symmetry $(2, 1)$ gains some shift gauge transformations with parameters from the BF sector.

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1 Introduction

Topological field theories [1, 2] are important in view of the fact that certain interacting, non-Abelian versions are related to a Poisson structure algebra [3] present in various versions of Poisson sigma models [4–10], which are known to be useful at the study of two-dimensional gravity [11–20] (for a detailed approach, see [21]). It is well known that pure three-dimensional gravity is just a BF theory. Moreover, in higher dimensions general relativity and supergravity in the Ashtekar formalism may also be formulated as topological BF theories with some extra constraints [22–25]. In view of these results, it is important to know the self-interactions in BF theories as well as the couplings between BF models and other theories. This problem has been considered in the literature in relation with self-interactions in various classes of BF models [26–33] and couplings to other (matter or gauge) fields [34–38] by using the powerful BRST cohomological reformulation of the problem of constructing consistent interactions within the Lagrangian [39, 40] or the Hamiltonian [41] setting, based on the computation of local BRST cohomology [42–44]. Other aspects concerning interacting, topological BF models can be found in [45] and [46].

On the other hand, tensor fields in "exotic" representations of the Lorentz group, characterized by a mixed Young symmetry type [47–53], held the attention lately on some important issues, like the dual formulation of field theories of spin two or higher [54–61], the impossibility of consistent cross-interactions in the dual formulation of linearized gravity [62], a Lagrangian first-order approach [63, 64] to some classes of massless or partially massive mixed symmetry type tensor gauge fields, suggestively resembling to the tetrad formalism of General Relativity, or the derivation of some exotic gravitational interactions [65, 66]. An important matter related to mixed symmetry type tensor fields is the study of their consistent interactions, among themselves as well as with other gauge theories [67–80].

The purpose of this paper is to investigate the consistent interactions in $D = 5$ between a massless tensor gauge field with the mixed symmetry of a two-column Young diagram of type $(2, 1)$ and an Abelian BF model with a maximal field spectrum (a scalar field, two sorts of one-forms, two types of two-forms and a three-form). It is worth mentioning the duality of a free massless tensor gauge field with the mixed symmetry $(2, 1)$ to the Pauli–Fierz theory in $D = 5$ dimensions. In view of this feature, we can state that our paper searches the consistent couplings in $D = 5$ between the dual formulation of linearized gravity and a topological BF model. Our analysis relies on the deformation of the solution to the master equation by means of cohomological techniques with the help of the local BRST cohomology. We mention that the self-interactions in the $(2, 1)$ sector have been investigated in [62] and the couplings in $D = 5$ that

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can be added to an Abelian BF model with a maximal field spectrum have been constructed in [32].

Under the hypotheses of analyticity in the coupling constant, spacetime locality, Lorentz covariance, and Poincaré invariance of the deformations, combined with the preservation of the number of derivatives on each field, we find a deformation of the solution to the master equation that provides nontrivial cross-couplings. The emerging Lagrangian action contains mixing-component terms of order one in the coupling constant that couple the massless tensor field with the mixed symmetry (2, 1) mainly to one of the two-forms and to the three-form from the BF sector. Also, it is interesting to note the appearance of some self-interactions in the BF sector at order two in the coupling constant that are strictly due to the presence of the tensor field with the mixed symmetry (2, 1) (they all vanish in its absence). The gauge transformations of all fields are deformed and, in addition, some of them include gauge parameters from the complementary sector. This is the first known case where the gauge transformations of the tensor field with the mixed symmetry (2, 1) do change with respect to the free ones (by shifts in some of the BF gauge parameters). The gauge algebra and the reducibility structure of the coupled model are strongly modified during the deformation procedure, becoming open and respectively on-shell, by contrast to the free theory, whose gauge algebra is Abelian and the reducibility relations hold off-shell. Our result is important because dual formulations of linearized gravity have proved to be extremely rigid in allowing consistent interactions to themselves as well as to many matter or gauge theories. Actually, we think that this is the first time when a massless tensor field with the mixed symmetry $(k, 1)$ allows consistent interactions that fulfill all the working hypotheses precisely in the dimension $D = k + 3$ where it becomes dual to the Pauli–Fierz theory.

2 The free theory: Lagrangian, gauge symmetries and BRST differential

The starting point is a free theory in $D = 5$, whose Lagrangian action is written as the sum between the Lagrangian action of an Abelian BF model with a maximal field spectrum (a single scalar field, φ, two types of one-forms, H^μ and V_μ, two kinds of two-forms, $B^{\mu\nu}$ and $\phi_{\mu\nu}$, and one three-form, $K^{\mu\nu\rho}$) and the Lagrangian action of a free, massless tensor field with the mixed symmetry (2, 1) $t_{\mu\nu|\alpha}$ (meaning it is antisymmetric in its first two indices $t_{\mu\nu|\alpha} = -t_{\nu\mu|\alpha}$ and fulfills the identity $t_{[\mu\nu|\alpha]} \equiv 0$)

$$S_0^{\mathrm{L}}\left[\Phi^{\alpha_0}\right] = \int d^5x \bigg[H^\mu \partial_\mu \varphi + \frac{1}{2} B^{\mu\nu} \partial_{[\mu} V_{\nu]} + \frac{1}{3} K^{\mu\nu\rho} \partial_{[\mu} \phi_{\nu\rho]} - \frac{1}{12}\left(F_{\mu\nu\rho|\alpha} F^{\mu\nu\rho|\alpha} - 3 F_{\mu\nu} F^{\mu\nu}\right)\bigg] \equiv \int d^5x \left(\mathcal{L}_0^{\mathrm{BF}} + \mathcal{L}_0^{\mathrm{t}}\right), \quad (1)$$

where we used the notations

$$\Phi^{\alpha_0} = \left(\varphi, H^\mu, V_\mu, B^{\mu\nu}, \phi_{\mu\nu}, K^{\mu\nu\rho}, t_{\mu\nu|\alpha}\right), \quad (2)$$

$$F_{\mu\nu\rho|\alpha} = \partial_{[\mu} t_{\nu\rho]|\alpha}, \qquad F_{\mu\nu} = \sigma^{\rho\alpha} F_{\mu\nu\rho|\alpha}. \quad (3)$$

Everywhere in this paper the notations $[\mu\nu\cdots\rho]$ and $(\mu\nu\cdots\rho)$ signify complete antisymmetry and respectively complete symmetry with respect to the (Lorentz) indices between brackets, with the conventions that the minimum number of terms is always used and the result is never divided by the number of terms. It is convenient to work with the Minkowski metric tensor of ‘mostly plus’ signature $\sigma_{\mu\nu} = \sigma^{\mu\nu} = \mathrm{diag}(- + + + +)$ and with the five-dimensional Levi–Civita symbol $\varepsilon^{\mu\nu\rho\lambda\sigma}$ defined according to the convention $\varepsilon^{01234} = -\varepsilon_{01234} = -1$.

Action (1) is found to be invariant under the gauge transformations

$$\delta_\Omega \varphi = 0, \qquad \delta_\Omega H^\mu = 2\partial_\nu \epsilon^{\mu\nu}, \quad (4)$$

$$\delta_\Omega V_\mu = \partial_\mu \epsilon, \qquad \delta_\Omega B^{\mu\nu} = -3\partial_\rho \epsilon^{\mu\nu\rho}, \quad (5)$$

$$\delta_\Omega \phi_{\mu\nu} = \partial_{[\mu} \xi_{\nu]}, \qquad \delta_\Omega K^{\mu\nu\rho} = 4\partial_\lambda \xi^{\mu\nu\rho\lambda}, \quad (6)$$

$$\delta_\Omega t_{\mu\nu|\alpha} = \partial_{[\mu} \theta_{\nu]\alpha} + \partial_{[\mu} \chi_{\nu]\alpha} - 2\partial_\alpha \chi_{\mu\nu}, \quad (7)$$

where all the gauge parameters are bosonic, with $\epsilon^{\mu\nu}$, $\epsilon^{\mu\nu\rho}$, $\xi^{\mu\nu\rho\lambda}$, and $\chi_{\mu\nu}$ completely antisymmetric and $\theta_{\mu\nu}$ symmetric. By Ω we denoted collectively all the gauge parameters as

$$\Omega^{\alpha_1} \equiv \left(\epsilon^{\mu\nu}, \epsilon, \epsilon^{\mu\nu\rho}, \xi_\mu, \xi^{\mu\nu\rho\lambda}, \theta_{\mu\nu}, \chi_{\mu\nu}\right). \quad (8)$$

The gauge transformations given by (4)–(7) are off-shell reducible of order three (the reducibility relations hold everywhere in the space of field history, and not only on the stationary surface of field equations). This means that:

1. There exist some transformations of the gauge parameters (8)

$$\Omega^{\alpha_1} \to \Omega^{\alpha_1} = \Omega^{\alpha_1}\left(\bar{\Omega}^{\alpha_2}\right), \quad (9)$$

such that the gauge transformations of all fields vanish strongly (first-order reducibility relations)

$$\delta_{\Omega(\bar{\Omega})} \Phi^{\alpha_0} = 0. \quad (10)$$

2. There exist some transformations of the first-order reducibility parameters $\bar{\Omega}^{\alpha_2}$

$$\bar{\Omega}^{\alpha_2} \to \bar{\Omega}^{\alpha_2} = \bar{\Omega}^{\alpha_2}\left(\check{\Omega}^{\alpha_3}\right), \quad (11)$$

such that the gauge parameters vanish strongly (second-order reducibility relations)

$$\Omega^{\alpha_1}\left(\bar{\Omega}^{\alpha_2}\left(\check{\Omega}^{\alpha_3}\right)\right)=0. \tag{12}$$

3. There exist some transformations of the second-order reducibility parameters $\check{\Omega}^{\alpha_3}$

$$\check{\Omega}^{\alpha_3}\rightarrow\check{\Omega}^{\alpha_3}=\check{\Omega}^{\alpha_3}\left(\hat{\Omega}^{\alpha_4}\right), \tag{13}$$

such that the first-order reducibility parameters vanish strongly (third-order reducibility relations)

$$\bar{\Omega}^{\alpha_2}\left(\check{\Omega}^{\alpha_3}\left(\hat{\Omega}^{\alpha_4}\right)\right)=0. \tag{14}$$

4. There is no nontrivial transformation of the third-order reducibility parameters $\hat{\Omega}^{\alpha_4}$ that annihilates all the second-order reducibility parameters

$$\check{\Omega}^{\alpha_3}\left(\hat{\Omega}^{\alpha_4}\right)=0 \quad\Leftrightarrow\quad \hat{\Omega}^{\alpha_4}=0. \tag{15}$$

This is indeed the case for the model under study. In this situation a complete set of first-order reducibility parameters $\bar{\Omega}^{\alpha_2}$ is given by

$$\bar{\Omega}^{\alpha_2}\equiv\left(\bar{\epsilon}^{\mu\nu\rho},\bar{\epsilon}^{\mu\nu\rho\lambda},\bar{\xi},\bar{\xi}^{\mu\nu\rho\lambda\sigma},\bar{\theta}_\mu\right), \tag{16}$$

and transformations (9) have the form

$$\begin{aligned}&\epsilon^{\mu\nu}\left(\bar{\Omega}^{\alpha_2}\right)=-3\partial_\rho\bar{\epsilon}^{\mu\nu\rho},\\&\epsilon\left(\bar{\Omega}^{\alpha_2}\right)=0,\qquad \epsilon^{\mu\nu\rho}\left(\bar{\Omega}^{\alpha_2}\right)=4\partial_\lambda\bar{\epsilon}^{\mu\nu\rho\lambda},\end{aligned} \tag{17}$$

$$\begin{aligned}&\xi_\mu\left(\bar{\Omega}^{\alpha_2}\right)=\partial_\mu\bar{\xi},\\&\xi^{\mu\nu\rho\lambda}\left(\bar{\Omega}^{\alpha_2}\right)=-5\partial_\sigma\bar{\xi}^{\mu\nu\rho\lambda\sigma},\end{aligned} \tag{18}$$

$$\begin{aligned}&\theta_{\mu\nu}\left(\bar{\Omega}^{\alpha_2}\right)=3\partial_{(\mu}\bar{\theta}_{\nu)},\\&\chi_{\mu\nu}\left(\bar{\Omega}^{\alpha_2}\right)=\partial_{[\mu}\bar{\theta}_{\nu]},\end{aligned} \tag{19}$$

with $\bar{\epsilon}^{\mu\nu\rho}$, $\bar{\epsilon}^{\mu\nu\rho\lambda}$, and $\bar{\xi}^{\mu\nu\rho\lambda\sigma}$ completely antisymmetric. Further, a complete set of second-order reducibility parameters $\check{\Omega}^{\alpha_3}$ can be taken as

$$\check{\Omega}^{\alpha_3}\equiv\left(\check{\epsilon}^{\mu\nu\rho\lambda},\check{\epsilon}^{\mu\nu\rho\lambda\sigma}\right), \tag{20}$$

and transformations (11) are

$$\begin{aligned}&\bar{\epsilon}^{\mu\nu\rho}\left(\check{\Omega}^{\alpha_3}\right)=4\partial_\lambda\check{\epsilon}^{\mu\nu\rho\lambda},\\&\bar{\epsilon}^{\mu\nu\rho\lambda}\left(\check{\Omega}^{\alpha_3}\right)=-5\partial_\sigma\check{\epsilon}^{\mu\nu\rho\lambda\sigma},\end{aligned} \tag{21}$$

$$\begin{aligned}&\bar{\xi}\left(\check{\Omega}^{\alpha_3}\right)=0,\qquad \bar{\xi}^{\mu\nu\rho\lambda\sigma}\left(\check{\Omega}^{\alpha_3}\right)=0,\\&\bar{\theta}_\mu\left(\check{\Omega}^{\alpha_3}\right)=0,\end{aligned} \tag{22}$$

where both $\check{\epsilon}^{\mu\nu\rho\lambda}$ and $\check{\epsilon}^{\mu\nu\rho\lambda\sigma}$ are some arbitrary, bosonic, completely antisymmetric tensors. Next, a complete set of third-order reducibility parameters $\hat{\Omega}^{\alpha_4}$ is represented by

$$\hat{\Omega}^{\alpha_4}\equiv\left(\hat{\epsilon}^{\mu\nu\rho\lambda\sigma}\right), \tag{23}$$

and transformations (13) can be chosen of the form

$$\begin{aligned}&\check{\epsilon}^{\mu\nu\rho\lambda}\left(\hat{\Omega}^{\alpha_4}\right)=-5\partial_\sigma\hat{\epsilon}^{\mu\nu\rho\lambda\sigma},\\&\check{\epsilon}^{\mu\nu\rho\lambda\sigma}\left(\hat{\Omega}^{\alpha_4}\right)=0,\end{aligned} \tag{24}$$

with $\hat{\epsilon}^{\mu\nu\rho\lambda\sigma}$ an arbitrary, completely antisymmetric tensor. Finally, it is easy to check (15). Indeed, we work in $D=5$, such that $\partial_\sigma\hat{\epsilon}^{\mu\nu\rho\lambda\sigma}=0$ implies $\hat{\epsilon}^{\mu\nu\rho\lambda\sigma}=\text{const.}$ Since $\hat{\epsilon}^{\mu\nu\rho\lambda\sigma}$ are arbitrary smooth functions that effectively depend on the spacetime coordinates, it follows that the only possible choice is $\hat{\epsilon}^{\mu\nu\rho\lambda\sigma}=0$.

We observe that the free theory under study is a usual linear gauge theory (its field equations are linear in the fields), whose generating set of gauge transformations is third-order reducible, such that we can define in a consistent manner its Cauchy order, which is found to be equal to five.

In order to construct the BRST symmetry of this free theory, we introduce the field/ghost and antifield spectra (2) and

$$\eta^{\alpha_1}=\left(C^{\mu\nu},\eta,\eta^{\mu\nu\rho},C_\mu,\mathcal{G}^{\mu\nu\rho\lambda},S_{\mu\nu},A_{\mu\nu}\right), \tag{25}$$

$$\eta^{\alpha_2}=\left(C^{\mu\nu\rho},\eta^{\mu\nu\rho\lambda},C,\mathcal{G}^{\mu\nu\rho\lambda\sigma},S_\mu\right), \tag{26}$$

$$\eta^{\alpha_3}=\left(C^{\mu\nu\rho\lambda},\eta^{\mu\nu\rho\lambda\sigma}\right),\qquad \eta^{\alpha_4}=\left(C^{\mu\nu\rho\lambda\sigma}\right), \tag{27}$$

$$\Phi^*_{\alpha_0}=\left(\varphi^*,H^*_\mu,V^{*\mu},B^*_{\mu\nu},\phi^{*\mu\nu},K^*_{\mu\nu\rho},t^{*\mu\nu|\alpha}\right), \tag{28}$$

$$\eta^*_{\alpha_1}=\left(C^*_{\mu\nu},\eta^*,\eta^*_{\mu\nu\rho},C^{*\mu},\mathcal{G}^*_{\mu\nu\rho\lambda},S^{*\mu\nu},A^{*\mu\nu}\right), \tag{29}$$

$$\eta^*_{\alpha_2}=\left(C^*_{\mu\nu\rho},\eta^*_{\mu\nu\rho\lambda},C^*,\mathcal{G}^*_{\mu\nu\rho\lambda\sigma},S^{*\mu}\right), \tag{30}$$

$$\eta^*_{\alpha_3}=\left(C^*_{\mu\nu\rho\lambda},\eta^*_{\mu\nu\rho\lambda\sigma}\right),\qquad \eta^*_{\alpha_4}=\left(C^*_{\mu\nu\rho\lambda\sigma}\right). \tag{31}$$

The fermionic ghosts (25) correspond to the bosonic gauge parameters (8), and therefore $C^{\mu\nu}$, $\eta^{\mu\nu\rho}$, $\mathcal{G}^{\mu\nu\rho\lambda}$, and $A_{\mu\nu}$ are completely antisymmetric and $S_{\mu\nu}$ is symmetric. The bosonic ghosts for ghosts (26) are respectively associated with the first-order reducibility parameters (16), such that $C^{\mu\nu\rho}$, $\eta^{\mu\nu\rho\lambda}$, and $\mathcal{G}^{\mu\nu\rho\lambda\sigma}$ are completely antisymmetric. Along the same line, the fermionic ghosts for ghosts for ghosts η^{α_3} from (27) correspond to the second-order reducibility parameters (20). As a consequence, the ghost fields $C^{\mu\nu\rho\lambda}$ and $\eta^{\mu\nu\rho\lambda\sigma}$ are again completely antisymmetric. Finally, the bosonic ghosts for ghosts for ghosts for ghosts η^{α_4} from (27) are associated with the third-order reducibility parameters (23), so $C^{\mu\nu\rho\lambda\sigma}$ is also completely antisymmetric. The star variables represent the antifields of the corresponding fields/ghosts. Their Grassmann parities are obtained via the usual rule $\varepsilon(\chi^*_\Delta)=(\varepsilon(\chi^\Delta)+1)\bmod 2$, where we employed the notations

$$\begin{aligned}&\chi^\Delta=\left(\Phi^{\alpha_0},\eta^{\alpha_1},\eta^{\alpha_2},\eta^{\alpha_3},\eta^{\alpha_4}\right),\\&\chi^*_\Delta=\left(\Phi^*_{\alpha_0},\eta^*_{\alpha_1},\eta^*_{\alpha_2},\eta^*_{\alpha_3},\eta^*_{\alpha_4}\right).\end{aligned} \tag{32}$$

It is understood that the antifields are endowed with the same symmetry/antisymmetry properties like those of the corresponding fields/ghosts.

Since both the gauge generators and the reducibility functions are field-independent, it follows that the BRST differential reduces to $s=\delta+\gamma$, where δ is the Koszul–Tate differential, and γ means the exterior longitudinal derivative. The Koszul–Tate differential is graded in terms of the antighost number (agh, agh $(\delta)=-1$, agh $(\gamma)=0$) and enforces a resolution of the algebra of smooth functions defined on the stationary surface of field equations for action (1), $C^{\infty}(\Sigma)$, $\Sigma: \delta S_0^{\mathrm{L}}/\delta\Phi^{\alpha_0}=0$. The exterior longitudinal derivative is graded in terms of the pure ghost number (pgh, pgh$(\gamma)=1$, pgh $(\delta)=0$) and is correlated with the original gauge symmetry via its cohomology in pure ghost number zero computed in $C^{\infty}(\Sigma)$, which is isomorphic to the algebra of physical observables for this free theory. These two degrees of generators (2) and (25)–(31) from the BRST complex are valued like

$$\begin{aligned}&\mathrm{pgh}\left(\Phi^{\alpha_0}\right)=0, \qquad \mathrm{pgh}\left(\eta^{\alpha_m}\right)=m,\\&\mathrm{pgh}\left(\Phi^*_{\alpha_0}\right)=\mathrm{pgh}\left(\eta^*_{\alpha_m}\right)=0,\end{aligned}\tag{33}$$

$$\begin{aligned}&\mathrm{agh}\left(\Phi^{\alpha_0}\right)=\mathrm{agh}\left(\eta^{\alpha_m}\right)=0, \qquad \mathrm{agh}\left(\Phi^*_{\alpha_0}\right)=1,\\&\mathrm{agh}\left(\eta^*_{\alpha_m}\right)=m+1,\end{aligned}\tag{34}$$

for $m=\overline{1,4}$. The actions of the differentials δ and γ on the above generators read

$$\left(\delta\Phi^{\alpha_0}=0,\ \delta\eta^{\alpha_m}=0,\ m=\overline{1,4}\right) \quad\Longleftrightarrow\quad \delta\chi^{\Delta}=0,\tag{35}$$

$$\begin{aligned}&\delta\varphi^*=\partial_\mu H^\mu, \qquad \delta H^*_\mu=-\partial_\mu\varphi,\\&\delta V^{*\mu}=-\partial_\nu B^{\mu\nu},\end{aligned}\tag{36}$$

$$\begin{aligned}&\delta B^*_{\mu\nu}=-\frac{1}{2}\partial_{[\mu}V_{\nu]}, \qquad \delta\phi^{*\mu\nu}=\partial_\rho K^{\mu\nu\rho},\\&\delta K^*_{\mu\nu\rho}=-\frac{1}{3}\partial_{[\mu}\phi_{\nu\rho]},\end{aligned}\tag{37}$$

$$\begin{aligned}&\delta t^{*\mu\nu|\alpha}=-\frac{1}{2}\partial_\rho\left(F^{\rho\mu\nu|\alpha}-\sigma^{\alpha[\mu}F^{\nu\rho]}\right),\\&\delta C^*_{\mu\nu}=\partial_{[\mu}H^*_{\nu]},\end{aligned}\tag{38}$$

$$\begin{aligned}&\delta\eta^*=-\partial_\mu V^{*\mu}, \qquad \delta\eta^*_{\mu\nu\rho}=\partial_{[\mu}B^*_{\nu\rho]},\\&\delta C^{*\mu}=2\partial_\nu\phi^{*\mu\nu},\end{aligned}\tag{39}$$

$$\begin{aligned}&\delta\mathcal{G}^*_{\mu\nu\rho\lambda}=\partial_{[\mu}K^*_{\nu\rho\lambda]}, \qquad \delta S^{*\mu\nu}=-\partial_\rho t^{*\rho(\mu|\nu)},\\&\delta A^{*\mu\nu}=3\partial_\rho t^{*\mu\nu|\rho},\end{aligned}\tag{40}$$

$$\begin{aligned}&\delta C^*_{\mu\nu\rho}=-\partial_{[\mu}C^*_{\nu\rho]}, \qquad \delta\eta^*_{\mu\nu\rho\lambda}=-\partial_{[\mu}\eta^*_{\nu\rho\lambda]},\\&\delta C^*=\partial_\mu C^{*\mu},\end{aligned}\tag{41}$$

$$\begin{aligned}&\delta\mathcal{G}^*_{\mu\nu\rho\lambda\sigma}=-\partial_{[\mu}\mathcal{G}^*_{\nu\rho\lambda\sigma]},\\&\delta S^{*\mu}=2\partial_\rho\left(3S^{*\rho\mu}+A^{*\rho\mu}\right)\equiv 2\partial_\rho C^{*\rho\mu},\end{aligned}\tag{42}$$

$$\begin{aligned}&\delta C^*_{\mu\nu\rho\lambda}=\partial_{[\mu}C^*_{\nu\rho\lambda]}, \qquad \delta\eta^*_{\mu\nu\rho\lambda\sigma}=\partial_{[\mu}\eta^*_{\nu\rho\lambda\sigma]},\\&\delta C^*_{\mu\nu\rho\lambda\sigma}=-\partial_{[\mu}C^*_{\nu\rho\lambda\sigma]},\end{aligned}\tag{43}$$

and respectively

$$\left(\gamma\Phi^*_{\alpha_0}=0,\ \gamma\eta^*_{\alpha_m}=0,\ m=\overline{1,4}\right) \quad\Longleftrightarrow\quad \gamma\chi^*_{\Delta}=0,\tag{44}$$

$$\gamma\varphi=0, \qquad \gamma H^\mu=2\partial_\nu C^{\mu\nu}, \qquad \gamma V_\mu=\partial_\mu\eta,\tag{45}$$

$$\begin{aligned}&\gamma B^{\mu\nu}=-3\partial_\rho\eta^{\mu\nu\rho}, \qquad \gamma\phi_{\mu\nu}=\partial_{[\mu}C_{\nu]},\\&\gamma K^{\mu\nu\rho}=4\partial_\lambda\mathcal{G}^{\mu\nu\rho\lambda},\end{aligned}\tag{46}$$

$$\begin{aligned}&\gamma t_{\mu\nu|\alpha}=\partial_{[\mu}S_{\nu]\alpha}+\partial_{[\mu}A_{\nu]\alpha}-2\partial_\alpha A_{\mu\nu},\\&\gamma C^{\mu\nu}=-3\partial_\rho C^{\mu\nu\rho},\end{aligned}\tag{47}$$

$$\gamma\eta=0, \qquad \gamma\eta^{\mu\nu\rho}=4\partial_\lambda\eta^{\mu\nu\rho\lambda}, \qquad \gamma C_\mu=\partial_\mu C,\tag{48}$$

$$\begin{aligned}&\gamma\mathcal{G}^{\mu\nu\rho\lambda}=-5\partial_\sigma\mathcal{G}^{\mu\nu\rho\lambda\sigma}, \qquad \gamma S_{\mu\nu}=3\partial_{(\mu}S_{\nu)},\\&\gamma A_{\mu\nu}=\partial_{[\mu}S_{\nu]},\end{aligned}\tag{49}$$

$$\begin{aligned}&\gamma C^{\mu\nu\rho}=4\partial_\lambda C^{\mu\nu\rho\lambda}, \qquad \gamma\eta^{\mu\nu\rho\lambda}=-5\partial_\sigma\eta^{\mu\nu\rho\lambda\sigma},\\&\gamma C=0,\end{aligned}\tag{50}$$

$$\begin{aligned}&\gamma\mathcal{G}^{\mu\nu\rho\lambda\sigma}=0, \qquad \gamma S_\mu=0,\\&\gamma C^{\mu\nu\rho\lambda}=-5\partial_\sigma C^{\mu\nu\rho\lambda\sigma},\end{aligned}\tag{51}$$

$$\gamma\eta^{\mu\nu\rho\lambda\sigma}=0, \qquad \gamma C^{\mu\nu\rho\lambda\sigma}=0.\tag{52}$$

The overall degree that grades the BRST complex is named ghost number (gh) and is defined like the difference between the pure ghost number and the antighost number, such that $\mathrm{gh}(\delta)=\mathrm{gh}(\gamma)=\mathrm{gh}(s)=1$.

The BRST symmetry admits a canonical action $s\cdot=(\cdot,\bar{S})$, where its canonical generator ($\mathrm{gh}(\bar{S})=0$, $\varepsilon(\bar{S})=0$) satisfies the classical master equation $(\bar{S},\bar{S})=0$. The symbol $(,)$ denotes the antibracket, defined by decreeing the fields/ghosts conjugated with the corresponding antifields. In the case of the free theory under discussion the solution to the master equation takes the form

$$\begin{aligned}\bar{S}=S_0^{\mathrm{L}}+\int d^5x\,\big[&2H^*_\mu\partial_\nu C^{\mu\nu}+V^{*\mu}\partial_\mu\eta\\&-3B^*_{\mu\nu}\partial_\rho\eta^{\mu\nu\rho}+\phi^{*\mu\nu}\partial_{[\mu}C_{\nu]}\\&+4K^*_{\mu\nu\rho}\partial_\lambda\mathcal{G}^{\mu\nu\rho\lambda}\\&+t^{*\mu\nu|\alpha}(\partial_{[\mu}S_{\nu]\alpha}+\partial_{[\mu}A_{\nu]\alpha}-2\partial_\alpha A_{\mu\nu})\\&-3C^*_{\mu\nu}\partial_\rho C^{\mu\nu\rho}+4\eta^*_{\mu\nu\rho}\partial_\lambda\eta^{\mu\nu\rho\lambda}\\&+C^{*\mu}\partial_\mu C-5\mathcal{G}^*_{\mu\nu\rho\lambda}\partial_\sigma\mathcal{G}^{\mu\nu\rho\lambda\sigma}\\&+3S^{*\mu\nu}\partial_{(\mu}S_{\nu)}+A^{*\mu\nu}\partial_{[\mu}S_{\nu]}+4C^*_{\mu\nu\rho}\partial_\lambda C^{\mu\nu\rho\lambda}\\&-5\eta^*_{\mu\nu\rho\lambda}\partial_\sigma\eta^{\mu\nu\rho\lambda\sigma}-5C^*_{\mu\nu\rho\lambda}\partial_\sigma C^{\mu\nu\rho\lambda\sigma}\big].\end{aligned}\tag{53}$$

The solution to the master equation encodes all the information on the gauge structure of a given theory. We remark that in our case solution (53) decomposes into terms with antighost numbers ranging from zero to four. Let us briefly recall the significance of the various terms present in the solution to the master equation. Thus, the part with the antighost number equal to zero is nothing but the Lagrangian action of the gauge model under study. The components of antighost number equal to one are always proportional with the gauge generators. If the gauge algebra were non-Abelian, then there would appear terms simultaneously linear in the antighost number two antifields and quadratic in the pure ghost number one ghosts. The absence of such terms in our case shows that the gauge transformations are Abelian. The terms from (53) with higher antighost numbers give us information on the reducibility functions. If the reducibility relations held on-shell, then there would appear components linear in the ghosts for ghosts (ghosts of pure ghost number strictly greater than one) and quadratic in the various antifields. Such pieces are not present in (53) since the reducibility relations (10), (12), and (14) hold off-shell. Other possible components in the solution to the master equation offer information on the higher-order structure functions related to the tensor gauge structure of the theory. There are no such terms in (53) as a consequence of the fact that all higher-order structure functions vanish for the theory under study.

3 Strategy

We begin with a "free" gauge theory, described by a Lagrangian action $S_0^{\mathrm{L}}[\Phi^{\alpha_0}]$, invariant under some gauge transformations

$$\delta_\epsilon \Phi^{\alpha_0} = Z^{\alpha_0}_{\alpha_1}\epsilon^{\alpha_1}, \qquad \frac{\delta S_0^{\mathrm{L}}}{\delta \Phi^{\alpha_0}} Z^{\alpha_0}_{\alpha_1} = 0, \tag{54}$$

and consider the problem of constructing consistent interactions among the fields Φ^{α_0} such that the couplings preserve both the field spectrum and the original number of gauge symmetries. This matter is addressed by means of reformulating the problem of constructing consistent interactions as a deformation problem of the solution to the master equation corresponding to the "free" theory [39, 40]. Such a reformulation is possible due to the fact that the solution to the master equation contains all the information on the gauge structure of the theory. If a consistent interacting gauge theory can be constructed, then the solution $\bar{S}$ to the master equation associated with the "free" theory, $(\bar{S}, \bar{S}) = 0$, can be deformed into a solution S,

$$\begin{aligned}\bar{S} \to S &= \bar{S} + \lambda S_1 + \lambda^2 S_2 + \cdots \\ &= \bar{S} + \lambda \int d^D x\, a + \lambda^2 \int d^D x\, b + \lambda^3 \int d^D x\, c \\ &\quad + \cdots\end{aligned} \tag{55}$$

of the master equation for the deformed theory

$$(S, S) = 0, \tag{56}$$

such that both the ghost and antifield spectra of the initial theory are preserved. The symbol $(,)$ denotes the antibracket. Equation (56) splits, according to the various orders in the coupling constant (or deformation parameter) λ, into the equivalent tower of equations

$$(\bar{S}, \bar{S}) = 0, \tag{57}$$

$$2(S_1, \bar{S}) = 0, \tag{58}$$

$$2(S_2, \bar{S}) + (S_1, S_1) = 0, \tag{59}$$

$$(S_3, \bar{S}) + (S_1, S_2) = 0, \tag{60}$$

$$2(S_4, \bar{S}) + (S_2, S_2) + 2(S_1, S_3) = 0 \tag{61}$$

$$\vdots$$

Equation (57) is fulfilled by hypothesis. The next one requires that the first-order deformation of the solution to the master equation, S_1, is a cocycle of the "free" BRST differential $s\cdot = (\cdot, \bar{S})$. However, only cohomologically nontrivial solutions to (58) should be taken into account, as the BRST-exact ones can be eliminated by (in general nonlinear) field redefinitions. This means that S_1 pertains to the ghost number zero cohomological space of s, $H^0(s)$, which is generically nonempty due to its isomorphism to the space of physical observables of the "free" theory. It has been shown in [39, 40] (on behalf of the triviality of the antibracket map in the cohomology of the BRST differential) that there are no obstructions in finding solutions to the remaining equations, namely, (59), (60) and so on. However, the resulting interactions may be nonlocal, and there might even appear obstructions if one insists on their locality. The analysis of these obstructions can be done with the help of cohomological techniques. As will be seen below, all the interactions in the case of the model under study turn out to be local.

4 Standard results

In the sequel we determine all consistent Lagrangian interactions that can be added to the free theory described by (1) and (4)–(7). This is done by means of solving the deformation equations (58)–(61), etc., with the help of specific cohomological techniques. The interacting theory and its gauge structure are then deduced from the analysis of the deformed solution to the master equation that is consistent to all orders in the deformation parameter.

For obvious reasons, we consider only analytical, local, Lorentz covariant, and Poincaré invariant deformations (i.e., we do not allow explicit dependence on the spacetime coordinates). The analyticity of deformations refers to the fact that the deformed solution to the master equation, (55), is analytical in the coupling constant λ and reduces to the original solution, (53), in the free limit $\lambda = 0$. In addition, we require that the overall interacting Lagrangian satisfies two further restrictions related to the derivative order of its vertices:

(i) The maximum derivative order of each interaction vertex is equal to two.
(ii) The differential order of each interacting field equation is equal to that of the corresponding free equation (meaning that at most one spacetime derivative can act on each field from the BF sector and at most two spacetime derivatives on the tensor field $t_{\mu\nu|\alpha}$).

If we make the notation $S_1 = \int d^5x\, a$, with a local, then (58) (which controls the first-order deformation) takes the local form

$$sa = \partial_\mu m^\mu, \qquad \mathrm{gh}\,(a) = 0, \qquad \varepsilon\,(a) = 0, \tag{62}$$

for some local m^μ. It shows that the nonintegrated density of the first-order deformation pertains to the local cohomology of s in ghost number zero, $a \in H^0\,(s|d)$, where d denotes the exterior spacetime differential. The solution to (62) is unique up to s-exact pieces plus divergences

$$a \rightarrow a + sb + \partial_\mu n^\mu. \tag{63}$$

If the general solution to (62) is trivial, $a = sb + \partial_\mu n^\mu$, then it can be made to vanish, $a = 0$.

In order to analyze (62) we develop a according to the antighost number

$$a = \sum_{i=0}^{I} a_i, \qquad \mathrm{agh}\,(a_i) = i,$$
$$\mathrm{gh}\,(a_i) = 0, \qquad \varepsilon\,(a_i) = 0, \tag{64}$$

and assume, without loss of generality, that the above decomposition stops at some finite value of I. This can be shown for instance like in [43] (Sect. 3), under the sole assumption that the interacting Lagrangian at order one in the coupling constant, a_0, has a finite, but otherwise arbitrary derivative order. Inserting (64) into (62) and projecting it on the various values of the antighost number, we obtain the tower of equations (equivalent to (62))

$$\gamma a_I = \partial_\mu \overset{(I)}{m}{}^{\mu}, \tag{65}$$

$$\delta a_I + \gamma a_{I-1} = \partial_\mu \overset{(I-1)}{m}{}^{\mu}, \tag{66}$$

$$\delta a_i + \gamma a_{i-1} = \partial_\mu \overset{(i-1)}{m}{}^{\mu}, \qquad I - 1 \geq i \geq 1, \tag{67}$$

for some local $(\overset{(i)}{m}{}^{\mu})_{i=\overline{0,I}}$. Equation (65) can always be replaced in strictly positive values of the antighost number by

$$\gamma a_I = 0, \qquad I > 0. \tag{68}$$

Due to the second-order nilpotency of γ ($\gamma^2 = 0$), the solution to (68) is unique up to γ-exact contributions

$$a_I \rightarrow a_I + \gamma b_I. \tag{69}$$

If a_I reduces only to γ-exact terms, $a_I = \gamma b_I$, then it can be made to vanish, $a_I = 0$. The nontriviality of the first-order deformation a is translated at its highest antighost number component into the requirement that $a_I \in H^I(\gamma)$, where $H^I(\gamma)$ denotes the cohomology of the exterior longitudinal derivative γ in pure ghost number equal to I. So, in order to solve (62) (equivalent with (68) and (66)–(67)), we need to compute the cohomology of γ, $H(\gamma)$, and, as will be made clear below, also the local homology of δ, $H(\delta|d)$.

From definitions (44)–(52) it is possible to show that $H(\gamma)$ is spanned by

$$F_{\bar{A}} = \big(\varphi, \partial_\mu H^\mu, \partial_{[\mu} V_{\nu]}, \partial_\mu B^{\mu\nu},$$
$$\partial_{[\mu}\phi_{\nu\rho]}, \partial_\mu K^{\mu\nu\rho}, R_{\mu\nu\rho|\alpha\beta}\big), \tag{70}$$

the antifields χ^*_Δ, and all of their spacetime derivatives as well as by the undifferentiated objects

$$\eta^{\bar{\Upsilon}} = \big(\eta, D_{\mu\nu\rho}, C, \mathcal{G}^{\mu\nu\rho\lambda\sigma}, S_\mu, \eta^{\mu\nu\rho\lambda\sigma}, C^{\mu\nu\rho\lambda\sigma}\big). \tag{71}$$

In (70) and (71) we respectively used the notations

$$R_{\mu\nu\rho|\alpha\beta} = -\frac{1}{2} F_{\mu\nu\rho|[\alpha,\beta]}, \qquad D_{\mu\nu\rho} = \partial_{[\mu} A_{\nu\rho]}, \tag{72}$$

with $f_{,\beta} \equiv \partial_\beta f$. It is useful to denote by $R_{\mu\nu|\alpha}$ and R_μ the trace and respectively double trace of $R_{\mu\nu\rho|\alpha\beta}$

$$R_{\mu\nu|\alpha} = \sigma^{\rho\beta} R_{\mu\nu\rho|\alpha\beta}, \qquad R_\mu = \sigma^{\rho\beta}\sigma^{\nu\alpha} R_{\mu\nu\rho|\alpha\beta}. \tag{73}$$

The spacetime derivatives (of any order) of all the objects from (71) are removed from $H(\gamma)$ since they are γ-exact. This can be seen directly from the last definition in (45), the last present in (47), the first from (49), the second in (50), the last from (51), and also using the relations

$$\partial_\alpha D_{\mu\nu\rho} = \gamma\left[-\frac{1}{2} F_{\mu\nu\rho|\alpha}\right],$$
$$\partial_\mu S_\nu = \gamma\left[\frac{1}{2}\left(\frac{1}{3} S_{\mu\nu} + A_{\mu\nu}\right)\right] \equiv \gamma\left[\frac{1}{2}\mathcal{C}_{\mu\nu}\right]. \tag{74}$$

Let $e^M(\eta^{\bar{\Upsilon}})$ be the elements with pure ghost number M of a basis in the space of polynomials in the objects (71). Then, the general solution to (68) takes the form (up, to trivial, γ-exact contributions)

$$a_I = \alpha_I\big([F_{\bar{A}}], [\chi^*_\Delta]\big) e^I\big(\eta^{\bar{\Upsilon}}\big), \tag{75}$$

where $\mathrm{agh}(\alpha_I) = I$ and $\mathrm{pgh}(e^I) = I$. The notation $f([q])$ means that f depends on q and its spacetime derivatives up to a finite order. The objects α_I (obviously nontrivial in $H^0(\gamma)$) will be called invariant 'polynomials'. They are true polynomials with respect to all variables (71) and their spacetime derivatives, excepting the undifferentiated scalar field φ, with respect to which α_I may be series. This is why we will keep the quotation marks around the word polynomial(s). The result that we can replace equation (65) with the less obvious one (68) for $I > 0$ is a nice consequence of the fact that the cohomology of the exterior spacetime differential is trivial in the space of invariant 'polynomials' in strictly positive antighost numbers. These results on $H(\gamma)$ can be synthesized in the following array:

BRST generator	pgh	Grassmann parity	Nontrivial object from $H(\gamma)$
χ^*_Δ	0	$(\varepsilon(\chi^\Delta)+1)\bmod 2$	$[\chi^*_\Delta]$
Φ^{α_0}	0	0	$[F_{\bar{A}}]$
η^{α_1}	1	1	$\eta,\ D_{\mu\nu\rho}\equiv\partial_{[\mu}A_{\nu\rho]}$
η^{α_2}	2	0	$C, \mathcal{G}^{\mu\nu\rho\lambda\sigma}, S_\mu$
η^{α_3}	3	1	$\eta^{\mu\nu\rho\lambda\sigma}$
η^{α_4}	4	0	$C^{\mu\nu\rho\lambda\sigma}$

(76)

where notations (2), (25)–(31), (33), and (70) should be taken into account.

Inserting (75) in (66) we obtain that a necessary (but not sufficient) condition for the existence of (nontrivial) solutions a_{I-1} is that the invariant 'polynomials' α_I are (nontrivial) objects from the local cohomology of Koszul–Tate differential $H(\delta|d)$ in antighost number $I > 0$ and in pure ghost number zero,

$$\delta\alpha_I = \partial_\mu \overset{(I-1)^\mu}{j}, \qquad \mathrm{agh}\left(\overset{(I-1)^\mu}{j}\right) = I-1, \qquad \mathrm{pgh}\left(\overset{(I-1)^\mu}{j}\right) = 0. \tag{77}$$

We recall that $H(\delta|d)$ is completely trivial in both strictly positive antighost *and* pure ghost numbers (for instance, see [42], Theorem 5.4, and [43]), so from now on it is understood that by $H(\delta|d)$ we mean the local cohomology of δ at pure ghost number zero. Using the fact that the free model under study is a linear gauge theory of Cauchy order equal to five and the general result from the literature [42, 43] according to which the local cohomology of the Koszul–Tate differential is trivial in antighost numbers strictly greater than its Cauchy order, we can state that

$$H_J(\delta|d) = 0 \quad \text{for all } J > 5, \tag{78}$$

where $H_J(\delta|d)$ represents the local cohomology of the Koszul–Tate differential in antighost number J. Moreover, it can be shown that if the invariant 'polynomial' α_J, with $\mathrm{agh}(\alpha_J) = J \geq 5$, is trivial in $H_J(\delta|d)$, then it can be taken to be trivial also in $H_J^{\mathrm{inv}}(\delta|d)$

$$\left(\alpha_J = \delta b_{J+1} + \partial_\mu \overset{(J)^\mu}{c},\ \mathrm{agh}(\alpha_J) = J \geq 5\right) \quad \Rightarrow \quad \alpha_J = \delta\beta_{J+1} + \partial_\mu \overset{(J)^\mu}{\gamma}, \tag{79}$$

with both β_{J+1} and $\overset{(J)^\mu}{\gamma}$ invariant 'polynomials'. Here, $H_J^{\mathrm{inv}}(\delta|d)$ denotes the invariant characteristic cohomology in antighost number J (the local cohomology of the Koszul–Tate differential in the space of invariant 'polynomials'). An element of $H_I^{\mathrm{inv}}(\delta|d)$ is defined via an equation like (77), but with the corresponding current an invariant 'polynomial'. This result together with (78) ensures that the entire invariant characteristic cohomology in antighost numbers strictly greater than five is trivial

$$H_J^{\mathrm{inv}}(\delta|d) = 0 \quad \text{for all } J > 5. \tag{80}$$

It is possible to show that no nontrivial representative of $H_J(\delta|d)$ or $H_J^{\mathrm{inv}}(\delta|d)$ for $J \geq 2$ is allowed to involve the spacetime derivatives of the fields [32] and [62]. Such a representative may depend at most on the undifferentiated scalar field φ. With the help of relations (35)–(43), it can be shown that $H^{\mathrm{inv}}(\delta|d)$ and $H(\delta|d)$ are spanned by the elements

agh	Nontrivial representative spanning $H_J^{\mathrm{inv}}(\delta|d)$	Grassmann parity
>5	None	–
5	$(W)_{\mu\nu\rho\lambda\sigma}$	1
4	$(W)_{\mu\nu\rho\lambda}, \eta^*_{\mu\nu\rho\lambda\sigma}$	0
3	$(W)_{\mu\nu\rho}, \eta^*_{\mu\nu\rho\lambda}, C^*, \mathcal{G}^*_{\mu\nu\rho\lambda\sigma}, S^{*\mu}$	1
2	$(W)_{\mu\nu}, \eta^*, \eta^*_{\mu\nu\rho}, C^{*\mu}, \mathcal{G}^*_{\mu\nu\rho\lambda}, S^{*\mu\nu}, A^{*\mu\nu}$	0

(81)

where

$$\begin{aligned}(W)_{\mu\nu\rho\lambda\sigma} &= \frac{dW}{d\varphi}C^*_{\mu\nu\rho\lambda\sigma} + \frac{d^2W}{d\varphi^2}\left(H^*_{[\mu}C^*_{\nu\rho\lambda\sigma]} + C^*_{[\mu\nu}C^*_{\rho\lambda\sigma]}\right) \\ &\quad + \frac{d^3W}{d\varphi^3}\left(H^*_{[\mu}H^*_\nu C^*_{\rho\lambda\sigma]} + H^*_{[\mu}C^*_{\nu\rho}C^*_{\lambda\sigma]}\right) \\ &\quad + \frac{d^4W}{d\varphi^4}H^*_{[\mu}H^*_\nu H^*_\rho C^*_{\lambda\sigma]} \\ &\quad + \frac{d^5W}{d\varphi^5}H^*_\mu H^*_\nu H^*_\rho H^*_\lambda H^*_\sigma,\end{aligned} \tag{82}$$

$$\begin{aligned}(W)_{\mu\nu\rho\lambda} &= \frac{dW}{d\varphi}C^*_{\mu\nu\rho\lambda} + \frac{d^2W}{d\varphi^2}\left(H^*_{[\mu}C^*_{\nu\rho\lambda]} + C^*_{[\mu\nu}C^*_{\rho\lambda]}\right) \\ &\quad + \frac{d^3W}{d\varphi^3}H^*_{[\mu}H^*_\nu C^*_{\rho\lambda]} \\ &\quad + \frac{d^4W}{d\varphi^4}H^*_\mu H^*_\nu H^*_\rho H^*_\lambda,\end{aligned} \tag{83}$$

$$(W)_{\mu\nu\rho} = \frac{dW}{d\varphi}C^*_{\mu\nu\rho} + \frac{d^2W}{d\varphi^2}H^*_{[\mu}C^*_{\nu\rho]}$$

$$+\frac{d^3W}{d\varphi^3}H^*_\mu H^*_\nu H^*_\rho, \tag{84}$$

$$(W)_{\mu\nu}=\frac{dW}{d\varphi}C^*_{\mu\nu}+\frac{d^2W}{d\varphi^2}H^*_\mu H^*_\nu, \tag{85}$$

whit $W=W(\varphi)$ an arbitrary, smooth function depending only on the undifferentiated scalar field φ.

In contrast to the spaces $(H_J(\delta|d))_{J\geq 2}$ and $(H^{\text{inv}}_J(\delta|d))_{J\geq 2}$, which are finite-dimensional, the cohomology $H_1(\delta|d)$ (known to be related to global symmetries and ordinary conservation laws) is infinite-dimensional since the theory is free. Fortunately, it will not be needed in the sequel.

The previous results on $H(\delta|d)$ and $H^{\text{inv}}(\delta|d)$ in strictly positive antighost numbers are important because they control the obstructions to removing the antifields from the first-order deformation. More precisely, we can successively eliminate all the pieces of antighost number strictly greater that five from the nonintegrated density of the first-order deformation by adding solely trivial terms, so we can take, without loss of nontrivial objects, the condition $I\leq 5$ into (64). In addition, the last representative is of the form (75), where the invariant 'polynomial' is necessarily a nontrivial object from $H^{\text{inv}}_5(\delta|d)$.

5 Computation of first-order deformation

In the case $I=5$ the nonintegrated density of the first-order deformation (see (64)) becomes

$$a=a_0+a_1+a_2+a_3+a_4+a_5. \tag{86}$$

We can further decompose a in a natural manner as a sum between two kinds of deformations

$$a=a^{\text{BF}}+a^{\text{int}}, \tag{87}$$

where a^{BF} contains only fields/ghosts/antifields from the BF sector and a^{int} describes the cross-interactions between the two theories.[1] The piece a^{BF} is completely known [32]. It is parameterized by seven smooth, but otherwise arbitrary functions of the undifferentiated scalar field, $(W_a(\varphi))_{a=\overline{1,6}}$ and $\bar{M}(\varphi)$. In the sequel we analyze the cross-interacting piece, a^{int}.

Due to the fact that a^{BF} and a^{int} involve different types of fields and that a^{BF} separately satisfies an equation of type (62), it follows that a^{int} is subject to the equation

$$sa^{\text{int}}=\partial^\mu m^{\text{int}}_\mu, \tag{88}$$

[1] Decomposition (87) does not include a component responsible for the self-interactions of the tensor field with the mixed symmetry (2, 1) since any such component has been proved in [62] to be trivial.

for some local current m^{int}_μ. In the sequel we determine the general solution to (88) that complies with all the hypotheses mentioned in the beginning of Sect. 4.

In agreement with (86), the general solution to the equation $sa^{\text{int}}=\partial^\mu m^{\text{int}}_\mu$ can be chosen to stop at antighost number $I=5$

$$a^{\text{int}}=a^{\text{int}}_0+a^{\text{int}}_1+a^{\text{int}}_2+a^{\text{int}}_3+a^{\text{int}}_4+a^{\text{int}}_5. \tag{89}$$

We will show in Appendices A, B and C that we can always take $a^{\text{int}}_5=a^{\text{int}}_4=a^{\text{int}}_3=0$ into decomposition (89), without loss of nontrivial contributions. Consequently, the first-order deformation of the solution to the master equation in the interacting case can be taken to stop at antighost number two

$$a^{\text{int}}=a^{\text{int}}_0+a^{\text{int}}_1+a^{\text{int}}_2, \tag{90}$$

where the components on the right-hand side of (90) are subject to (68) and (66)–(67) for $I=2$.

The piece a^{int}_2 as solution to (68) for $I=2$ has the general form expressed by (75) for $I=2$, with α_2 from $H^{\text{inv}}_2(\delta|d)$. Looking at formula (76) and also at relation (81) in antighost number two and requiring that a^{int}_2 mixes BRST generators from the BF and (2, 1) sectors, we get that the most general solution to (68) for $I=2$ reads[2]

$$\begin{aligned}a^{\text{int}}_2={}&q_9\eta^{*\mu\nu\rho}\eta D_{\mu\nu\rho}+\left(q_{10}\tilde{\mathcal{G}}^{*\mu}+q_{11}C^{*\mu}\right)S_\mu\\&+q_{12}A^{*\mu\nu}\eta\tilde{D}_{\mu\nu}\\&+\frac{q_{13}}{2}\tilde{\eta}^{*\mu\nu}\sigma^{\alpha\beta}\tilde{D}_{\mu\alpha}\tilde{D}_{\nu\beta}+S^*(k_1C+k_2\tilde{\mathcal{G}}),\end{aligned} \tag{91}$$

where all quantities denoted by q or k are some real, arbitrary constants.

In the above and from now on we will use a compact writing in terms of the Hodge duals

$$\tilde{\Psi}^{\nu_1\ldots\nu_j}=\frac{1}{(5-j)!}\varepsilon^{\nu_1\ldots\nu_j\mu_1\ldots\mu_{5-j}}\Psi_{\mu_1\ldots\mu_{5-j}}. \tag{92}$$

Consequently $\tilde{\eta}^{*\mu\nu}$, $\tilde{\mathcal{G}}^{*\varepsilon}$ and $\tilde{\mathcal{G}}_\rho$ are the Hodge duals of $\eta^*_{\rho\lambda\sigma}$, $\mathcal{G}^*_{\mu\nu\rho\lambda}$, and respectively $\mathcal{G}^{\mu\nu\lambda\sigma}$.

Substituting (91) in (66) for $I=2$ and using definitions (35)–(52), we determine the solution a^{int}_1 under the form

$$a^{\text{int}}_1=-3q_9B^{*\mu\nu}\left(V^\rho D_{\mu\nu\rho}+\frac{1}{2}\eta F_{\mu\nu}\right)$$

[2] In principle, one can add to a^{int}_2 the terms $(\tilde{M}_2)^{\mu\nu\rho}\eta D_{\mu\nu\rho}+\frac{1}{2}(M_3)^{\mu\nu}\sigma^{\alpha\beta}\tilde{D}_{\mu\alpha}\tilde{D}_{\nu\beta}$, where $(\tilde{M}_2)^{\mu\nu\rho}$ is the Hodge dual of an expression similar to (85) with $W(\varphi)\rightarrow M_2(\varphi)$, and $(M_3)^{\mu\nu}$ reads as in (85) with $W(\varphi)\rightarrow M_3(\varphi)$. Both M_2 and M_3 are some arbitrary, real, smooth functions depending on the undifferentiated scalar field. It can be shown that the above terms finally lead to trivial interactions, so they can be removed from the first-order deformation.

$$-\left(\frac{q_{10}}{2}\tilde{K}^{*\mu\nu}+q_{11}\phi^{*\mu\nu}\right)A_{\mu\nu}$$
$$-3q_{12}t^{*\mu\nu|\rho}\left(V_\rho\tilde{D}_{\mu\nu}+\frac{1}{2}\eta\tilde{F}_{\mu\nu|\rho}\right)$$
$$+\frac{q_{13}}{2}\tilde{B}^{*\mu\nu\rho}\sigma^{\alpha\beta}\tilde{F}_{\mu\alpha|\rho}\tilde{D}_{\nu\beta}$$
$$-2t^*_\mu\left(k_1C^\mu-\frac{k_2}{5}\tilde{\mathcal{G}}^\mu\right)+\bar{a}_1^{\text{int}}, \tag{93}$$

where $\tilde{F}_{\lambda\mu|\alpha}$ is the Hodge dual of $F^{\nu\rho\sigma}_{\ \ \ \ |\alpha}$ defined in (3) with respect to its first three indices

$$\tilde{F}_{\lambda\mu|\alpha}=\frac{1}{3!}\varepsilon_{\lambda\mu\nu\rho\sigma}F^{\nu\rho\sigma}_{\ \ \ \ |\alpha}. \tag{94}$$

In the last formulas $\tilde{K}_{\lambda\sigma}$ is the dual of the three-form $K^{\mu\nu\rho}$ from action (1), $\tilde{B}^{*\rho\lambda\sigma}$ and $\tilde{K}^{*\lambda\sigma}$ represent the duals of the antifields $B^*_{\mu\nu}$ and respectively $K^*_{\mu\nu\rho}$ from (28).

In the above $\bar{a}_1^{\text{int}}$ is the solution to the homogeneous equation (68) in antighost number one, meaning that $\bar{a}_1^{\text{int}}$ is a nontrivial object from $H(\gamma)$ in pure ghost number one and in antighost number one. It is useful to decompose $\bar{a}_1^{\text{int}}$ like in (C.4)

$$\bar{a}_1^{\text{int}}=\hat{a}_1^{\text{int}}+\check{a}_1^{\text{int}}, \tag{95}$$

with $\hat{a}_1^{\text{int}}$ the solution to (68) for $I=1$ that ensures the consistency of a_1^{int} in antighost number zero, namely the existence of a_0^{int} as solution to (67) for $i=1$ with respect to the terms from a_1^{int} containing the constants of type q or k, and $\check{a}_1^{\text{int}}$ the solution to (68) for $I=1$ that is independently consistent in antighost number zero

$$\delta\check{a}_1^{\text{int}}=-\gamma\check{c}_0+\partial_\mu\check{m}_0^\mu. \tag{96}$$

With the help of definitions (35)–(52) and taking into account decomposition (C.4), we infer by direct computation

$$\delta a_1^{\text{int}}=\delta\left[\hat{a}_1^{\text{int}}+\left(2k_1K^{*\mu\nu\rho}+\frac{k_2}{30}\tilde{K}^{*\mu\nu\rho}\right)D_{\mu\nu\rho}\right]$$
$$+\gamma c_0+\partial_\lambda j_0^\lambda+\chi_0, \tag{97}$$

where

$$c_0=-\check{c}_0+\frac{q_{13}}{16}\tilde{V}^{\mu\nu\rho\lambda}\sigma^{\alpha\beta}\tilde{F}_{\mu\alpha|\rho}\tilde{F}_{\nu\beta|\lambda}$$
$$-\left(k_1\phi^{\mu\nu}-\frac{k_2}{20}\tilde{K}^{\mu\nu}\right)F_{\mu\nu}, \tag{98}$$

$$\chi_0=-3q_9\left[\left(\partial^{[\mu}V^{\nu]}\right)V^\rho D_{\mu\nu\rho}+V^\mu\left(\partial^\nu\eta\right)F_{\mu\nu}-V^\mu\eta R_\mu\right]$$
$$+\frac{1}{18}\left(q_{10}\tilde{\phi}^{\mu\nu\rho}+6q_{11}K^{\mu\nu\rho}\right)D_{\mu\nu\rho}$$
$$-\frac{3q_{12}}{4}\left[\partial_\rho\left(F^{\rho\mu\nu|\alpha}-\sigma^{\alpha[\mu}F^{\nu\rho]}\right)\right]$$
$$\times\left(2V_\alpha\tilde{D}_{\mu\nu}+\eta\tilde{F}_{\mu\nu|\alpha}\right)$$
$$+\frac{q_{13}}{8}\varepsilon^{\mu\nu\rho\lambda\sigma}\sigma^{\alpha\beta}\tilde{R}_{\mu\alpha|\lambda\sigma}\left(2V_\rho\tilde{D}_{\nu\beta}+\eta\tilde{F}_{\nu\beta|\rho}\right), \tag{99}$$

and j_0^λ are some local currents. In the above $\tilde{V}^{\mu\nu\rho\lambda}$ and $\tilde{\phi}^{\mu\nu\lambda}$ represent the Hodge duals of the one-form V_σ and respectively of the two-form $\phi_{\rho\sigma}$ from (2) and $\tilde{R}_{\lambda\sigma|\alpha\beta}$ is nothing but the Hodge dual of the tensor $R^{\mu\nu\rho}_{\ \ \ \ |\alpha\beta}$ defined in (72) with respect to its first three indices, namely

$$\tilde{R}_{\lambda\sigma|\alpha\beta}=\frac{1}{3!}\varepsilon_{\lambda\sigma\mu\nu\rho}R^{\mu\nu\rho}_{\ \ \ \ |\alpha\beta}. \tag{100}$$

Inspecting (97), we observe that (67) for $i=1$ possesses solutions if and only if χ_0 expressed by (99) is γ-exact modulo d. A straightforward analysis of χ_0 shows that this is not possible unless

$$q_9=q_{10}=q_{11}=q_{12}=q_{13}=0. \tag{101}$$

Now, we insert conditions (101) in (91) and identify the most general form of the first-order deformation in the interacting sector at antighost number two

$$a_2^{\text{int}}=S^*\left(k_1C+k_2\tilde{\mathcal{G}}\right). \tag{102}$$

The same conditions replaced in (97) enable us to write

$$\hat{a}_1^{\text{int}}=-\left(2k_1K^{*\mu\nu\rho}+\frac{k_2}{30}\tilde{\phi}^{*\mu\nu\rho}\right)D_{\mu\nu\rho}. \tag{103}$$

Introducing (103) in (95) and then the resulting result together with (101) in (93), we obtain

$$a_1^{\text{int}}=-2t^*_\mu\left(k_1C^\mu+\frac{k_2}{5}\tilde{\mathcal{G}}^\mu\right)$$
$$-\left(2k_1K^{*\mu\nu\rho}+\frac{k_2}{30}\tilde{\phi}^{*\mu\nu\rho}\right)D_{\mu\nu\rho}+\check{a}_1^{\text{int}}. \tag{104}$$

Next, we determine $\check{a}_1^{\text{int}}$ as the solution to the homogeneous (68) for $I=1$ that is independently consistent in antighost number zero, i.e. satisfies (96). According to (75) for $I=1$ the general solution to (68) for $I=1$ has the form

$$\check{a}_1^{\text{int}}=t^{*\mu\nu|\rho}\left(L_{\mu\nu|\rho}\eta+L^{\alpha\beta\gamma}_{\mu\nu|\rho}D_{\alpha\beta\gamma}\right)$$
$$+\left(V^*_\alpha M^\alpha_{\mu\nu\rho}+\varphi^*M_{\mu\nu\rho}+H^*_\alpha\bar{M}^\alpha_{\mu\nu\rho}\right.$$
$$\left.+B^*_{\alpha\beta}M^{\alpha\beta}_{\mu\nu\rho}+\phi^*_{\alpha\beta}\bar{M}^{\alpha\beta}_{\mu\nu\rho}+K^*_{\alpha\beta\gamma}M^{\alpha\beta\gamma}_{\mu\nu\rho}\right)D^{\mu\nu\rho}$$
$$+\left(V^*_\alpha N^\alpha+\varphi^*N+H^*_\alpha\bar{N}^\alpha+B^*_{\alpha\beta}N^{\alpha\beta}\right.$$
$$\left.+\phi^*_{\alpha\beta}\bar{N}^{\alpha\beta}+K^*_{\alpha\beta\gamma}N^{\alpha\beta\gamma}\right)\eta, \tag{105}$$

where all the quantities denoted by L, M, N, $\bar{M}$, or $\bar{N}$ are bosonic, gauge-invariant tensors, and therefore they may depend only on $F_{\bar{A}}$ given in (70) and their spacetime derivatives. The functions $L_{\mu\nu|\rho}$ and $L^{\alpha\beta\gamma}_{\mu\nu|\rho}$ exhibit the mixed symmetry (2, 1) with respect to their lower indices and, in addition, $L^{\alpha\beta\gamma}_{\mu\nu|\rho}$ is completely antisymmetric with respect to its upper indices. The remaining functions, M, $\bar{M}$, N, and $\bar{N}$, are separately antisymmetric (where appropriate) in their upper and respectively lower indices.

In order to determine all possible solutions (105) we demand that $\check{a}^{\rm int}_1$ mixes the BF and (2, 1) sectors and (for the first time) explicitly implement the assumption on the derivative order of the interacting Lagrangian discussed in the beginning of Sect. 4 and structured in requirements (i) and (ii). Because all the terms involving the functions N or $\bar{N}$ contain only BRST generators from the BF sector, it follows that each such function must contain at least one tensor $R_{\mu\nu\rho|\alpha\beta}$ defined in (72), with F as in (3). The corresponding terms from $\check{a}^{\rm int}_1$, if consistent, would produce an interacting Lagrangian that does not agree with requirement (ii) with respect to the BF fields and therefore we must take

$$N^{\alpha} = N = \bar{N}^{\alpha} = N^{\alpha\beta} = \bar{N}^{\alpha\beta} = N^{\alpha\beta\gamma} = 0. \tag{106}$$

In the meantime, requirement (ii) also restricts all the functions M and $\bar{M}$ to be derivative-free. Since the undifferentiated scalar field is the only element among $F_{\bar{A}}$ and their spacetime derivatives that contains no derivatives, it follows that all M and $\bar{M}$ may depend at most on φ. Due to the fact that we work in $D = 5$ and taking into account the various antisymmetry properties of these functions, it follows that the only eligible representations are

$$M^{\alpha}_{\mu\nu\rho} = M_{\mu\nu\rho} = \bar{M}^{\alpha}_{\mu\nu\rho} = 0, \tag{107}$$

$$M^{\alpha\beta}_{\mu\nu\rho} = U_{13}\varepsilon^{\alpha\beta}{}_{\mu\nu\rho}, \qquad \bar{M}^{\alpha\beta}_{\mu\nu\rho} = U_{14}\varepsilon^{\alpha\beta}{}_{\mu\nu\rho},$$

$$M^{\alpha\beta\gamma}_{\mu\nu\rho} = \frac{1}{6}U_{15}\delta^{\alpha}_{[\mu}\delta^{\beta}_{\nu}\delta^{\gamma}_{\rho]}, \tag{108}$$

with U_{13}, U_{14}, and U_{15} some real, smooth functions of φ. The same observation stands for $L_{\mu\nu|\rho}$ and $L^{\alpha\beta\gamma}_{\mu\nu|\rho}$, so their tensorial behavior can only be realized via some constant Lorentz tensors. Nevertheless, there is no such constant tensor in $D = 5$ with the required mixed symmetry properties, and hence we must put

$$L_{\mu\nu|\rho} = 0, \qquad L^{\alpha\beta\gamma}_{\mu\nu|\rho} = 0. \tag{109}$$

Inserting results (106)–(109) in (105), it follows that the most general (nontrivial) solution to equation (68) for $I = 1$ that complies with all the working hypotheses, including that on the differential order of the interacting Lagrangian, is given by

$$\check{a}^{\rm int}_1 = \varepsilon^{\mu\nu\rho\lambda\sigma}\left(U_{13}B^{*}_{\mu\nu} + U_{14}\phi^{*}_{\mu\nu}\right)D_{\rho\lambda\sigma} + U_{15}K^{*\mu\nu\rho}D_{\mu\nu\rho}. \tag{110}$$

By acting with δ on (110) and using definitions (35)–(52) we infer

$$\begin{aligned}\delta\check{a}^{\rm int}_1 = {}& \gamma\left[\left(-3U_{14}\tilde{K}^{\mu\nu} + 2U_{15}\phi^{\mu\nu}\right)F_{\mu\nu}\right] \\ & + \partial_{\alpha}\left(\varepsilon^{\mu\nu\rho\lambda\alpha}U_{13}V_{\mu}D_{\nu\rho\lambda}\right. \\ & \left. - \varepsilon_{\mu\nu\rho\lambda\sigma}U_{14}K^{\alpha\mu\nu}D^{\rho\lambda\sigma} + U_{14}\phi_{\mu\nu}D^{\alpha\mu\nu}\right) \\ & + \varepsilon_{\mu\nu\rho\lambda\sigma}\left[-\left(\partial^{\mu}U_{13}\right)V^{\nu} + \left(\partial_{\alpha}U_{14}\right)K^{\alpha\mu\nu}\right]D^{\rho\lambda\sigma} \\ & - \left(\partial_{\mu}U_{15}\right)\phi_{\nu\rho}D^{\mu\nu\rho} \\ & + 2F^{\mu\nu}\left(6U_{14}\partial_{[\mu}\tilde{\mathcal{G}}_{\nu]} - U_{15}\partial_{[\mu}C_{\nu]}\right).\end{aligned} \tag{111}$$

Comparing (111) with (96), we conclude that function U_{13} reduces to a real constant and meanwhile functions U_{14} and U_{15} must vanish

$$U_{13} = u_{13}, \qquad U_{14} = 0 = U_{15}, \tag{112}$$

so (110) becomes

$$\check{a}^{\rm int}_1 = \varepsilon^{\mu\nu\rho\lambda\sigma}u_{13}B^{*}_{\mu\nu}D_{\rho\lambda\sigma}, \tag{113}$$

which produces trivial deformations because it is a trivial element from $H_1(\delta|d)$:

$$\check{a}^{\rm int}_1 = \delta\left(\varepsilon^{\mu\nu\rho\lambda\sigma}u_{13}\eta^{*}_{\mu\nu\rho}A_{\lambda\sigma}\right) + \partial_{\mu}\left(\varepsilon^{\mu\nu\rho\lambda\sigma}u_{13}B^{*}_{\nu\rho}A_{\lambda\sigma}\right) \tag{114}$$

and by further taking

$$\check{a}^{\rm int}_1 = 0. \tag{115}$$

As a consequence, we can safely take the nontrivial part of the first-order deformation in the interaction sector in antighost number one, (104), of the form

$$a^{\rm int}_1 = -2t^{*}_{\mu}\left(k_1C^{\mu} + \frac{k_2}{5}\tilde{\mathcal{G}}^{\mu}\right) - \left(2k_1K^{*\mu\nu\rho} + \frac{k_2}{30}\tilde{\phi}^{*\mu\nu\rho}\right)D_{\mu\nu\rho}. \tag{116}$$

In addition, (115) leads to

$$\check{c}_0 = 0, \qquad \check{m}^{\mu}_0 = 0 \tag{117}$$

in (96). Replacing now (101) and (117) in (97), we are able to identify the piece of antighost number zero from the first-order deformation in the interacting sector as

$$a^{\rm int}_0 = \left(k_1\phi^{\mu\nu} - \frac{k_2}{20}\tilde{K}^{\mu\nu}\right)F_{\mu\nu} + \bar{a}^{\rm int}_0, \tag{118}$$

where $\bar{a}^{\rm int}_0$ is the solution to the 'homogeneous' equation in antighost number zero

$$\gamma\bar{a}^{\rm int}_0 = \partial_{\mu}\bar{m}^{\mu}_0. \tag{119}$$

We will prove in Appendix D that the only solution to (119) that satisfies all our working hypotheses, including that on the derivative order of the interacting Lagrangian, is $\bar{a}_0^{\rm int} = 0$, such that the nontrivial part of the first-order deformation in the interaction sector in antighost number zero reads

$$a_0^{\rm int} = \left(k_1 \phi^{\mu\nu} - \frac{k_2}{20}\tilde{K}^{\mu\nu}\right) F_{\mu\nu}. \tag{120}$$

The main conclusion of this section is that the general form of the first-order deformation of the solution to the master equation as solution to (58) for the model under study is expressed by

$$S_1 = \int d^5x \left(a^{\rm BF} + a^{\rm int}\right), \tag{121}$$

where $a^{\rm BF}$ can be found in [32] and

$$\begin{aligned} a^{\rm int} &= a_0^{\rm int} + a_1^{\rm int} + a_2^{\rm int} \\ &= S^*(k_1 C + k_2 \tilde{\mathcal{G}}) - 2t_\mu^* \left(k_1 C^\mu + \frac{k_2}{5}\tilde{\mathcal{G}}^\mu\right) \\ &\quad - \left(2k_1 K^{*\mu\nu\rho} + \frac{k_2}{30}\tilde{\phi}^{*\mu\nu\rho}\right) D_{\mu\nu\rho} \\ &\quad + \left(k_1 \phi^{\mu\nu} - \frac{k_2}{20}\tilde{K}^{\mu\nu}\right) F_{\mu\nu}. \end{aligned} \tag{122}$$

It is now clear that the first-order deformation is parameterized by seven arbitrary, smooth functions of the undifferentiated scalar field ($(W_a(\varphi))_{a=\overline{1,6}}$ and $\bar{M}(\varphi)$ corresponding to $a^{\rm BF}$ and by two arbitrary, real constants (k_1 and k_2 from $a^{\rm int}$). We will see in the next section that the consistency of the deformed solution to the master equation in order two in the coupling constant will restrict these functions and constants to satisfy some specific equations.

6 Computation of higher-order deformations

With the first-order deformation at hand, in the sequel we determine the higher-order deformations of the solution to the master equation, governed by (59)–(61), etc., which comply with our working hypotheses.

In the first step we approach the second-order deformation, S_2, as (nontrivial) solution to (59). If we denote by Δ the nonintegrated density of the antibracket (S_1, S_1) and by b the nonintegrated density associated with S_2,

$$(S_1, S_1) = \int d^5x\, \Delta, \qquad S_2 = \int d^5x\, b, \tag{123}$$

then (59) takes the local form

$$\Delta + 2sb = \partial_\mu n^\mu, \tag{124}$$

with n^μ a local current. By direct computation it follows that Δ decomposes as

$$\Delta = \Delta^{\rm BF} + \Delta^{\rm int}, \tag{125}$$

where $\Delta^{\rm BF}$ involves only BRST generators from the BF sector and each term from $\Delta^{\rm int}$ depends simultaneously on the BRST generators of both sectors (BF and mixed symmetry (2, 1)), such that $\Delta^{\rm int}$ couples the two theories. Consequently, decomposition (125) induces a similar one at the level of the second-order deformation

$$b = b^{\rm BF} + b^{\rm int} \tag{126}$$

and (124) becomes equivalent to two equations, one for the BF sector and the other for the interacting sector

$$\Delta^{\rm BF} + 2sb^{\rm BF} = \partial^\mu n_\mu^{\rm BF}, \tag{127}$$

$$\Delta^{\rm int} + 2sb^{\rm int} = \partial^\mu n_\mu^{\rm int}. \tag{128}$$

Equation (127) has been completely solved in [32], where it was shown that it possesses only the trivial solution

$$b^{\rm BF} = 0 \tag{129}$$

and, in addition, the seven functions $(W_a)_{a=\overline{1,6}}$ and $\bar{M}(\varphi)$ that parameterize $a^{\rm BF}$ are subject to the following equations:

$$\frac{d\bar{M}(\varphi)}{d\varphi} W_1(\varphi) = 0, \qquad W_1(\varphi) W_2(\varphi) = 0, \tag{130}$$

$$W_1(\varphi)\frac{dW_2(\varphi)}{d\varphi} - 3W_2(\varphi)W_3(\varphi) + 6W_5(\varphi)W_6(\varphi) = 0, \tag{131}$$

$$W_2(\varphi)W_3(\varphi) + W_5(\varphi)W_6(\varphi) = 0, \tag{132}$$

$$W_1(\varphi)\frac{dW_6(\varphi)}{d\varphi} + 3W_3(\varphi)W_6(\varphi) - 6W_2(\varphi)W_4(\varphi) = 0, \tag{133}$$

$$W_1(\varphi)W_6(\varphi) = 0, \qquad W_2(\varphi)W_4(\varphi) + W_3(\varphi)W_6(\varphi) = 0, \tag{134}$$

$$W_2(\varphi)W_5(\varphi) = 0, \qquad W_4(\varphi)W_6(\varphi) = 0. \tag{135}$$

Now, we investigate the latter equation, (128). By direct computation $\Delta^{\rm int}$ can be brought to the form

$$\Delta^{\rm int} = s\left[-3\left(k_1\phi_{\mu\nu} - \frac{k_2}{20}\tilde{K}_{\mu\nu}\right)\left(k_1\phi^{\mu\nu} - \frac{k_2}{20}\tilde{K}^{\mu\nu}\right)\right] + \bar{\Delta}^{\rm int} + \partial^\mu \bar{n}_\mu^{\rm int}, \tag{136}$$

where $\bar{n}_\mu^{\rm int}$ is a local current and

$$\bar{\Delta}^{\rm int} = \sum_{i=1}^{3}\sum_{p=0}^{3} \frac{d^p \bar{Y}^{(i)}}{d\varphi^p} \bar{X}_p^{(i)}. \tag{137}$$

In $\bar{\Delta}^{\rm int}$ we used the notations

$$\begin{aligned} \bar{Y}^{(1)} &= k_1 W_3 + \frac{k_2}{60} W_5, \\ \bar{Y}^{(2)} &= k_1 W_4 + \frac{k_2}{2\cdot 5!} W_3, \end{aligned} \tag{138}$$

$$\bar{Y}^{(3)} = k_1 W_6 + \frac{k_2}{5!} W_2, \tag{139}$$

and the polynomials $\bar{X}_p^{(i)}$ are listed in Appendix E (see formulas (E.1)–(E.12)). It can be shown that (137) cannot be written as a s-exact modulo d element from local functions and therefore it must vanish

$$\bar{\Delta}^{\rm int} = 0, \tag{140}$$

which further restricts the functions and constants that parameterize the first-order deformation to obey the supplementary equations

$$\begin{aligned} k_1 W_3 + \frac{k_2}{60} W_5 &= 0, \\ k_1 W_4 + \frac{k_2}{2\cdot 5!} W_3 &= 0, \end{aligned} \tag{141}$$

$$k_1 W_6 + \frac{k_2}{5!} W_2 = 0. \tag{142}$$

As a consequence, the consistency of the first-order deformation at order two in the coupling constant (the existence of local solutions to (59)) on the one hand restricts the functions and constants that parameterize S_1 to fulfill (130)–(135) and (141)–(142) and, on the other hand, enables us (via formulas (123), (126), (128), (129), (136), and (140)) to infer the second-order deformation as

$$S_2 = S_2^{\rm int} = \int d^5x \left[\frac{3}{2}\left(k_1 \phi_{\mu\nu} - \frac{k_2}{20}\tilde{K}_{\mu\nu}\right) \times \left(k_1 \phi^{\mu\nu} - \frac{k_2}{20}\tilde{K}^{\mu\nu}\right)\right]. \tag{143}$$

In the second step we solve the equation that governs the third-order deformation, namely, (60). If we make the notations

$$\begin{aligned} (S_1, S_2) &= \int d^5x\, \Lambda, \\ S_3 &= \int d^5x\, c, \end{aligned} \tag{144}$$

then (60) takes the local form

$$\Lambda + sc = \partial_\mu p^\mu, \tag{145}$$

with p^μ a local current. By direct computation we obtain

$$\Lambda = \partial_\mu \bar{p}^\mu + \sum_{i=1}^{3}\sum_{p=0}^{2} \frac{d^p \bar{Y}^{(i)}}{d\varphi^p} U_p^{(i)}, \tag{146}$$

where $\bar{p}^\mu$ is a local current and the functions $U_p^{(i)}$ appearing in the right-hand side of (146) are listed in Appendix E (see formulas (E.13)–(E.21)). Taking into account the result that the functions and constants that parameterize both the first- and second-order deformations satisfy (130)–(135) and (141)–(142) and comparing (146) with (145), it results that the third-order deformation can be chosen to be completely trivial

$$S_3 = 0. \tag{147}$$

Related to the equation that governs the fourth-order deformation, namely, (61), we have that

$$2(S_1, S_3) + (S_2, S_2) = 0. \tag{148}$$

From (148) and (61) we find that S_4 is completely trivial

$$S_4 = 0. \tag{149}$$

Along a similar line, it can be shown that all the remaining higher-order deformations S_k ($k \geq 5$) can be taken to vanish

$$S_k = 0, \quad k \geq 5. \tag{150}$$

The main conclusion of this section is that the deformed solution to the master equation for the model under study, which is consistent to all orders in the coupling constant, can be taken as

$$S = \bar{S} + \lambda S_1 + \lambda^2 S_2, \tag{151}$$

where $\bar{S}$ reads as in (53), S_1 is given in (121) with $a^{\rm int}$ of the form (122), and S_2 is expressed by (143). It represents the most general solution that complies with all our working hypotheses (see the discussion from the beginning of Sect. 4). We cannot stress enough that the (seven) functions and (two) constants that parameterize the fully deformed solution to the master equation are no longer independent. They must obey (130)–(135) and (141)–(142).

7 The coupled theory: Lagrangian and gauge structure

In this section we start from the concrete form of (151) and identify the entire gauge structure of the Lagrangian model that describes all consistent interactions in $D=5$ between the BF theory and the massless tensor field $t_{\mu\nu|\alpha}$. To this end we recall the discussion from the end of Sect. 2 related to the relationship between the gauge structure of a given Lagrangian field theory and various terms of definite antighost number present in the solution of the master equation. Of course, we assume that the functions $(W_a)_{a=\overline{1,6}}$, $\bar{M}$ together with the constants k_1 and k_2 satisfy (130)–(135) and (141)–(142). The analysis of solutions that are interesting from the point of view of cross-couplings (at least one of the constants k_1 and k_2 is nonvanishing) is done in Sect. 8.

The piece of antighost number zero from (151) provides nothing but the Lagrangian action of the interacting theory

$$\begin{aligned} S^{\mathrm{L}}\left[\Phi^{\alpha_0}\right] = \int d^5x \Bigg\{ & H_\mu \partial^\mu \varphi + \frac{1}{2} B^{\mu\nu} \partial_{[\mu} V_{\nu]} \\ & + \frac{1}{3} K^{\mu\nu\rho} \partial_{[\mu} \phi_{\nu\rho]} \\ & + \lambda \bigg[W_1 V_\mu H^\mu + W_2 B_{\mu\nu} \phi^{\mu\nu} \\ & - W_3 \phi_{[\mu\nu} V_{\rho]} K^{\mu\nu\rho} + \bar{M}(\varphi) \\ & + \varepsilon^{\alpha\beta\gamma\delta\varepsilon} \bigg(9 W_4 V_\alpha \tilde{K}_{\beta\gamma} \tilde{K}_{\delta\varepsilon} \\ & + \frac{1}{4} W_5 V_\alpha \phi_{\beta\gamma} \phi_{\delta\varepsilon} + W_6 B_{\alpha\beta} K_{\gamma\delta\varepsilon} \bigg) \bigg] \\ & - \frac{1}{12} \left(F_{\mu\nu\rho|\alpha} F^{\mu\nu\rho|\alpha} - 3 F_{\mu\nu} F^{\mu\nu} \right) \\ & + \lambda \left(k_1 \phi^{\mu\nu} - \frac{k_2}{20} \tilde{K}^{\mu\nu} \right) \\ & \times \left[F_{\mu\nu} + \frac{3\lambda}{2} \left(k_1 \phi_{\mu\nu} - \frac{k_2}{20} \tilde{K}_{\mu\nu} \right) \right] \Bigg\}, \end{aligned} \tag{152}$$

where Φ^{α_0} is the field spectrum (2). The terms of antighost number one from the deformed solution of the master equation, generically written as $\Phi^*_{\alpha_0} Z^{\alpha_0}{}_{\alpha_1} \eta^{\alpha_1}$, allow the identification of the gauge transformations of action (152) via replacing the ghosts η^{α_1} with the gauge parameters Ω^{α_1}

$$\bar{\delta}_\Omega \Phi^{\alpha_0} = Z^{\alpha_0}{}_{\alpha_1} \Omega^{\alpha_1}. \tag{153}$$

In our case, taking into account formula (151) and maintaining the notation (8) for the gauge parameters, we find the concrete form of the deformed gauge transformations:

$$\bar{\delta}_\Omega \varphi = -\lambda W_1 \epsilon, \tag{154}$$

$$\begin{aligned} \bar{\delta}_\Omega H^\mu = & 2 D_\nu \epsilon^{\mu\nu} + \lambda \left(\frac{dW_1}{d\varphi} H^\mu - 3 \frac{dW_3}{d\varphi} K^{\mu\nu\rho} \phi_{\nu\rho} \right) \epsilon \\ & - 3\lambda \frac{dW_2}{d\varphi} \phi_{\nu\rho} \epsilon^{\mu\nu\rho} \\ & + 2\lambda \left(\frac{dW_2}{d\varphi} B^{\mu\nu} - 3 \frac{dW_3}{d\varphi} K^{\mu\nu\rho} V_\rho \right) \xi_\nu \\ & + 12\lambda \frac{dW_3}{d\varphi} V_\nu \phi_{\rho\lambda} \xi^{\mu\nu\rho\lambda} \\ & + 2\lambda \frac{dW_6}{d\varphi} B^{\mu\nu} \varepsilon_{\nu\alpha\beta\gamma\delta} \xi^{\alpha\beta\gamma\delta} \\ & + 3\lambda K^{\mu\nu\rho} \bigg(4 \frac{dW_4}{d\varphi} V_\nu \varepsilon_{\rho\alpha\beta\gamma\delta} \xi^{\alpha\beta\gamma\delta} \\ & - \frac{dW_6}{d\varphi} \varepsilon_{\nu\rho\alpha\beta\gamma} \epsilon^{\alpha\beta\gamma} \bigg) \\ & + \lambda \varepsilon^{\mu\nu\rho\lambda\sigma} \bigg[\frac{1}{4} \frac{dW_4}{d\varphi} \varepsilon_{\nu\rho\alpha\beta\gamma} K^{\alpha\beta\gamma} \varepsilon_{\lambda\sigma\alpha'\beta'\gamma'} K^{\alpha'\beta'\gamma'} \epsilon \\ & - \frac{dW_5}{d\varphi} \phi_{\nu\rho} \left(V_\lambda \xi_\sigma - \frac{1}{4} \phi_{\lambda\sigma} \epsilon \right) \bigg], \end{aligned} \tag{155}$$

$$\bar{\delta}_\Omega V_\mu = \partial_\mu \epsilon - 2\lambda W_2 \xi_\mu - 2\lambda \varepsilon_{\mu\nu\rho\lambda\sigma} W_6 \xi^{\nu\rho\lambda\sigma}, \tag{156}$$

$$\begin{aligned} \bar{\delta}_\Omega B^{\mu\nu} = & -3 \partial_\rho \epsilon^{\mu\nu\rho} - 2\lambda W_1 \epsilon^{\mu\nu} \\ & + 6\lambda W_3 \left(2 \phi_{\rho\lambda} \xi^{\mu\nu\rho\lambda} + K^{\mu\nu\rho} \xi_\rho \right) \\ & + \lambda \big(12 W_4 K^{\mu\nu\rho} \varepsilon_{\rho\alpha\beta\gamma\delta} \xi^{\alpha\beta\gamma\delta} \\ & - W_5 \varepsilon^{\mu\nu\rho\lambda\sigma} \phi_{\rho\lambda} \xi_\sigma \big), \end{aligned} \tag{157}$$

$$\begin{aligned} \bar{\delta}_\Omega \phi_{\mu\nu} = & D^{(-)}_{[\mu} \xi_{\nu]} + 3\lambda \left(W_3 \phi_{\mu\nu} \epsilon - 2 W_4 V_{[\mu} \varepsilon_{\nu]\alpha\beta\gamma\delta} \xi^{\alpha\beta\gamma\delta} \right) \\ & + 3\lambda \varepsilon_{\mu\nu\rho\lambda\sigma} \bigg(2 W_4 K^{\rho\lambda\sigma} \epsilon + W_6 \epsilon^{\rho\lambda\sigma} \\ & - \frac{k_2}{180} \partial^{[\rho} \chi^{\lambda\sigma]} \bigg), \end{aligned} \tag{158}$$

$$\begin{aligned} \bar{\delta}_\Omega K^{\mu\nu\rho} = & 4 D^{(+)}_\lambda \xi^{\mu\nu\rho\lambda} - 3\lambda \left(W_2 \epsilon^{\mu\nu\rho} + W_3 K^{\mu\nu\rho} \epsilon \right) \\ & - \lambda \varepsilon^{\mu\nu\rho\lambda\sigma} W_5 \left(V_\lambda \xi_\sigma - \frac{1}{2} \phi_{\lambda\sigma} \epsilon \right) \\ & - 2\lambda k_1 \partial^{[\mu} \chi^{\nu\rho]}, \end{aligned} \tag{159}$$

$$\begin{aligned} \bar{\delta}_\Omega t_{\mu\nu|\alpha} = & \partial_{[\mu} \theta_{\nu]\alpha} + \partial_{[\mu} \chi_{\nu]\alpha} - 2 \partial_\alpha \chi_{\mu\nu} \\ & + \lambda k_1 \sigma_{\alpha[\mu} \xi_{\nu]} - \frac{\lambda k_2}{5!} \sigma_{\alpha[\mu} \varepsilon_{\nu]\beta\gamma\delta\varepsilon} \xi^{\beta\gamma\delta\varepsilon}, \end{aligned} \tag{160}$$

where, in addition, we used the notations

$$\begin{aligned} D_\nu &= \partial_\nu - \lambda \frac{dW_1}{d\varphi} V_\nu, \\ D^{(\pm)}_\nu &= \partial_\nu \pm 3\lambda W_3 V_\nu. \end{aligned} \tag{161}$$

We observe that the cross-interaction terms,

$$\lambda \left(k_1 \phi^{\mu\nu} - \frac{k_2}{20} \tilde{K}^{\mu\nu} \right) F_{\mu\nu},$$

are only of order one in the deformation parameter and couple the tensor field $t_{\lambda\mu|\alpha}$ to the two-form $\phi_{\mu\nu}$ and to the three-form $K^{\mu\nu\rho}$ from the BF sector. Also, it is interesting to see that the interaction components

$$\frac{3\lambda^2}{2}\left(k_1\phi^{\mu\nu}-\frac{k_2}{20}\tilde{K}^{\mu\nu}\right)\left(k_1\phi_{\mu\nu}-\frac{k_2}{20}\tilde{K}_{\mu\nu}\right),$$

which describe self-interactions in the BF sector, are strictly due to the presence of the tensor $t_{\lambda\mu|\alpha}$ (in its absence $k_1 = k_2 = 0$, so they would vanish). The gauge transformations of the BF fields $\phi_{\mu\nu}$ and $K^{\mu\nu\rho}$ are deformed in such a way to include gauge parameters from the (2, 1) sector. Related to the other BF fields, φ, H^μ, V_μ, and $B^{\mu\nu}$, their gauge transformations are also modified with respect to the free theory, but only with terms specific to the BF sector. A remarkable feature is that the gauge transformations of the tensor $t_{\lambda\mu|\alpha}$ are modified by shift terms in some of the gauge parameters from the BF sector.

From the components of higher antighost number present in (151) we read the entire gauge structure of the interacting theory: the commutators among the deformed gauge transformations (154)–(160), and hence the properties of the deformed gauge algebra, their associated higher-order structure functions, and also the new reducibility functions and relations together with their properties. (The reducibility order itself of the interacting theory is not modified by the deformation procedure and remains equal to that of the free model, namely, three.) We do not give here the concrete form of all these deformed structure functions, which is analyzed in detail in Appendix F, but only briefly discuss their main properties by contrast to the gauge features of the free theory (see Sect. 2).

The nonvanishing commutators among the deformed gauge transformations result from the terms quadratic in the ghosts with pure ghost number one present in (151). Since their form can be generically written as $\frac{1}{2}(\eta^*_{\alpha_1}C^{\alpha_1}{}_{\beta_1\gamma_1}-\frac{1}{2}\Phi^*_{\alpha_0}\Phi^*_{\beta_0}M^{\alpha_0\beta_0}_{\beta_1\gamma_1})\eta^{\beta_1}\eta^{\gamma_1}$, it follows that the commutators among the deformed gauge transformations only close on-shell (on the stationary surface of the deformed field equations)

$$[\bar{\delta}_{\Omega_1},\bar{\delta}_{\Omega_2}]\Phi^{\alpha_0}=\bar{\delta}_{\Omega}\Phi^{\alpha_0}+M^{\alpha_0\beta_0}_{\Omega}\frac{\delta S^{\mathrm{L}}}{\delta\Phi^{\beta_0}}. \tag{162}$$

Here, $\delta S^{\mathrm{L}}/\delta\Phi^{\beta_0}$ stand for the Euler–Lagrange (EL) derivatives of the interacting action (152), Ω_1 and Ω_2 represent two independent sets of gauge parameters of type (8), and Ω is a quadratic combination of Ω_1 and Ω_2. The exact form of the corresponding commutators is included in the Appendix F (see formulas (F.3)–(F.9)). In conclusion, the gauge algebra corresponding to the interacting theory is open (the commutators among the deformed gauge transformations only close on-shell), by contrast to the free theory, where the gauge algebra is Abelian.

The first-order reducibility functions and relations follow from the terms linear in the ghosts for ghosts appearing in (151). Because they can be generically set in the form $(\eta^*_{\alpha_1}Z^{\alpha_1}{}_{\alpha_2}+\frac{1}{2}\Phi^*_{\alpha_0}\Phi^*_{\beta_0}C^{\alpha_0\beta_0}_{\alpha_2})\eta^{\alpha_2}$, it follows that if we transform the gauge parameters Ω^{α_1} in terms of the first-order reducibility parameters $\bar{\Omega}^{\alpha_2}$ as in

$$\Omega^{\alpha_1}\to\Omega^{\alpha_1}=Z^{\alpha_1}{}_{\alpha_2}\bar{\Omega}^{\alpha_2}, \tag{163}$$

then the transformed gauge transformations (153) of all fields vanish on-shell

$$\delta_{\Omega(\bar{\Omega})}\Phi^{\alpha_0}\equiv Z^{\alpha_0}{}_{\alpha_1}Z^{\alpha_1}{}_{\alpha_2}\bar{\Omega}^{\alpha_2}=C^{\alpha_0\beta_0}_{\bar{\Omega}}\frac{\delta S^{\mathrm{L}}}{\delta\Phi^{\beta_0}}\approx 0. \tag{164}$$

Along the same line, the second-order reducibility functions and relations are given by the terms linear in the ghosts for ghosts for ghosts appearing in (151), which can be generically written as $(\eta^*_{\alpha_2}Z^{\alpha_2}{}_{\alpha_3}-\eta^*_{\alpha_1}\Phi^*_{\beta_0}C^{\alpha_1\beta_0}_{\alpha_3}+\cdots)\eta^{\alpha_3}$. Consequently, if we transform the first-order reducibility parameters $\bar{\Omega}^{\alpha_2}$ in terms of the second-order reducibility parameters $\check{\Omega}^{\alpha_3}$ as in

$$\bar{\Omega}^{\alpha_2}\to\bar{\Omega}^{\alpha_2}=Z^{\alpha_2}{}_{\alpha_3}\check{\Omega}^{\alpha_3}, \tag{165}$$

then the transformed gauge parameters (163) vanish on-shell

$$\begin{aligned}\Omega^{\alpha_1}\big(\bar{\Omega}^{\alpha_2}\big(\check{\Omega}^{\alpha_3}\big)\big)&\equiv Z^{\alpha_1}{}_{\alpha_2}Z^{\alpha_2}{}_{\alpha_3}\check{\Omega}^{\alpha_3}\\&=C^{\alpha_1\beta_0}_{\check{\Omega}}\frac{\delta S^{\mathrm{L}}}{\delta\Phi^{\beta_0}}\approx 0.\end{aligned} \tag{166}$$

Finally, the third-order reducibility functions and relations are withdrawn from the terms linear in the ghosts for ghosts for ghosts for ghosts from (151), which have the generic form $(\eta^*_{\alpha_3}Z^{\alpha_3}{}_{\alpha_4}+\eta^*_{\alpha_2}\Phi^*_{\beta_0}C^{\alpha_2\beta_0}_{\alpha_4}+\cdots)\eta^{\alpha_4}$, such that if we transform the second-order reducibility parameters $\check{\Omega}^{\alpha_3}$ in terms of the third-order reducibility parameters $\hat{\Omega}^{\alpha_4}$ as in

$$\check{\Omega}^{\alpha_3}\to\check{\Omega}^{\alpha_3}=Z^{\alpha_3}{}_{\alpha_4}\hat{\Omega}^{\alpha_4}, \tag{167}$$

then the transformed first-order reducibility parameters (165) again vanish on-shell

$$\begin{aligned}\bar{\Omega}^{\alpha_2}\big(\check{\Omega}^{\alpha_3}\big(\hat{\Omega}^{\alpha_4}\big)\big)&\equiv Z^{\alpha_2}{}_{\alpha_3}Z^{\alpha_3}{}_{\alpha_4}\hat{\Omega}^{\alpha_4}\\&=C^{\alpha_2\beta_0}_{\check{\Omega}}\frac{\delta S^{\mathrm{L}}}{\delta\Phi^{\beta_0}}\approx 0.\end{aligned} \tag{168}$$

In the above the notations Ω^{α_1}, $\bar{\Omega}^{\alpha_2}$, $\check{\Omega}^{\alpha_3}$, and $\hat{\Omega}^{\alpha_4}$ are the same from the free case, namely (8), (16), (20), and (23), while the BRST generators are structured according to formulas (25)–(31). It is now clear that the reducibility relations associated with the interacting model ((164), (166), and (168)) only hold on-shell, by contrast to those corresponding to the free theory ((10), (12), and respectively (14)), which hold off-shell. Their concrete form is detailed in Appendix F.

8 Some solutions to the consistency equations

Equations (130)–(135) and (141)–(142), required by the consistency of the first-order deformation, possess the following classes of solutions, interesting from the point of view of cross-couplings between the BF field sector and the tensor field with the mixed symmetry (2, 1).

I. The real constants k_1 and k_2 are arbitrary ($k_1^2 + k_2^2 > 0$), functions $\bar{M}$ and W_2 are some arbitrary, real, smooth functions of the undifferentiated scalar field, and

$$W_1(\varphi) = W_3(\varphi) = W_4(\varphi) = W_5(\varphi) = 0, \tag{169}$$

$$W_6(\varphi) = -\frac{k_2}{5!k_1} W_2(\varphi). \tag{170}$$

The above formulas allow one to infer directly the solution in the general case $k_2 = 0$. This class of solutions can be equivalently reformulated as: the real constants k_1 and k_2 are arbitrary ($k_1^2 + k_2^2 > 0$), functions $\bar{M}$ and W_6 are some arbitrary, real, smooth functions of the undifferentiated scalar field, and

$$W_1(\varphi) = W_3(\varphi) = W_4(\varphi) = W_5(\varphi) = 0, \tag{171}$$

$$W_2(\varphi) = -\frac{5!k_1}{k_2} W_6(\varphi). \tag{172}$$

The last formulas are useful at writing down the solution in the particular case $k_1 = 0$.

II. The real constants k_1 and k_2 are arbitrary ($k_1^2 + k_2^2 > 0$), functions $\bar{M}$ and W_5 are some arbitrary, real, smooth functions of the undifferentiated scalar field, and

$$W_1(\varphi) = W_2(\varphi) = W_6(\varphi) = 0, \tag{173}$$

$$W_3(\varphi) = -\frac{k_2}{60k_1} W_5(\varphi), \qquad W_4(\varphi) = \left(\frac{k_2}{5!k_1}\right)^2 W_5(\varphi). \tag{174}$$

The above formulas allow one to infer directly the solution in the general case $k_2 = 0$. This class of solutions can be equivalently reformulated as: the real constants k_1 and k_2 are arbitrary ($k_1^2 + k_2^2 > 0$), functions $\bar{M}$ and W_4 are some arbitrary, real, smooth functions of the undifferentiated scalar field, and

$$W_1(\varphi) = W_2(\varphi) = W_6(\varphi) = 0, \tag{175}$$

$$W_3(\varphi) = -2 \cdot 5!\frac{k_1}{k_2} W_4(\varphi), \qquad W_5(\varphi) = \left(\frac{5!k_1}{k_2}\right)^2 W_4(\varphi). \tag{176}$$

The last formulas are useful at writing down the solution in the particular case $k_1 = 0$.

III. The real constants k_1 and k_2 are arbitrary ($k_1^2 + k_2^2 > 0$), functions W_1 and W_5 are some arbitrary, real, smooth functions of the undifferentiated scalar field, and

$$W_2(\varphi) = W_6(\varphi) = \bar{M}(\varphi) = 0, \tag{177}$$

$$W_3(\varphi) = -\frac{k_2}{60k_1} W_5(\varphi), \qquad W_4(\varphi) = \left(\frac{k_2}{5!k_1}\right)^2 W_5(\varphi). \tag{178}$$

The above formulas allow one to infer directly the solution in the general case $k_2 = 0$. This class of solutions can be equivalently reformulated as: the real constants k_1 and k_2 are arbitrary ($k_1^2 + k_2^2 > 0$), functions W_1 and W_4 are some arbitrary, real, smooth functions of the undifferentiated scalar field, and

$$W_2(\varphi) = W_6(\varphi) = \bar{M}(\varphi) = 0, \tag{179}$$

$$W_3(\varphi) = -2 \cdot 5!\frac{k_1}{k_2} W_4(\varphi), \qquad W_5(\varphi) = \left(\frac{5!k_1}{k_2}\right)^2 W_4(\varphi). \tag{180}$$

The last formulas are useful at writing down the solution in the particular case $k_1 = 0$.

For all classes of solutions the emerging interacting theories display the following common features:

1. There appear nontrivial cross-couplings between the BF fields and the tensor field with the mixed symmetry (2, 1).
2. The gauge transformations are modified with respect to those of the free theory and the gauge algebras become open (only close on-shell).
3. The first-order reducibility functions are changed during the deformation process and the first-order reducibility relations take place on-shell.

Nevertheless, there appear the following differences between the above classes of solutions at the level of the higher-order reducibility:

(a) For class I the second-order reducibility functions are modified with respect to the free ones and the corresponding reducibility relations take place on-shell. The third-order reducibility functions remain those from the free case and hence the associated reducibility relations hold off-shell.

(b) For class II both the second- and third-order reducibility functions remain those from the free case and hence the associated reducibility relations hold off-shell.

(c) For class III all the second- and third-order reducibility functions are deformed and the corresponding reducibility relations only close on-shell.

9 Conclusion

The most important conclusion of this paper is that under the hypotheses of analyticity in the coupling constant, space-time locality, Lorentz covariance, and Poincaré invariance of the deformations, combined with the preservation of the number of derivatives on each field, the dual formulation of linearized gravity in $D=5$ allows for the first time non-trivial couplings to another theory, namely with a topological BF model, whose field spectrum consists in a scalar field, two sorts of one-forms, two types of two-forms, and a three-form. The deformed Lagrangian contains mixing-component terms of order one in the deformation parameter that couple the massless tensor field with the mixed symmetry $(2,1)$ mainly to one of the two-forms and to the three-form from the BF sector. There appear some self-interactions in the BF sector at order two in the coupling constant that are strictly due to the presence of the tensor field with the mixed symmetry $(2,1)$. One of the striking features of the deformed model is that the gauge transformations of all fields are deformed. This is the first case where the gauge transformations of the tensor field with the mixed symmetry $(2,1)$ do change with respect to the free ones (by shifts in some of the BF gauge parameters). All the ingredients of the gauge structure are modified by the deformation procedure: the gauge algebra becomes open and the reducibility relations hold on-shell.

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Appendix A: No-go result for $I=5$ in a^{int}

In agreement with (86), the general solution to the equation $sa^{\mathrm{int}}=\partial^{\mu}m_{\mu}^{\mathrm{int}}$ can be chosen to stop at antighost number $I=5$:

$$a^{\mathrm{int}}=a_0^{\mathrm{int}}+a_1^{\mathrm{int}}+a_2^{\mathrm{int}}+a_3^{\mathrm{int}}+a_4^{\mathrm{int}}+a_5^{\mathrm{int}}, \tag{A.1}$$

where the components on the right-hand side of (A.1) are subject to (68) and (66)–(67) for $I=5$.

The piece a_5^{int} as solution to (68) for $I=5$ has the general form expressed by (75) for $I=5$, with α_5 from $H_5^{\mathrm{inv}}(\delta|d)$. According to (81) at antighost number five, it follows that $H_5^{\mathrm{inv}}(\delta|d)$ is spanned by the generic representatives (82). Since a_5^{int} should effectively mix the BF and the $(2,1)$ tensor field sectors in order to produce cross-couplings and (82) involves only BF generators, it follows that one should retain from the basis elements $e^5(\eta^{\bar{\Upsilon}})$ only the objects containing at least one ghost from the $(2,1)$ tensor field sector, namely $D_{\mu\nu\rho}$ or S_{μ}. Recalling that we work precisely in $D=5$, we obtain that the general solution to (68) for $I=5$ reduces to

$$\begin{aligned}a_5^{\mathrm{int}}={}&\frac{1}{3!}\big((\tilde{U}_1)C+(\tilde{U}_2)\tilde{\mathcal{G}}\big)\tilde{D}_{\mu\alpha}\tilde{D}^{\alpha\beta}\tilde{D}_{\beta\nu}\sigma^{\mu\nu}\\&+\frac{1}{2}\big((\tilde{U}_3)\eta S^{\mu}-(\tilde{U}_4)D^{\mu\nu\rho}\tilde{D}_{\nu\alpha}\tilde{D}_{\rho\beta}\sigma^{\alpha\beta}\big)S_{\mu}.\end{aligned} \tag{A.2}$$

Each tilde object from the right-hand side of (A.2) means the Hodge dual of the corresponding non-tilde element, defined in general by formula (92). The elements $\tilde{U}$ are dual to $(U)_{\mu_1\ldots\mu_5}$ as in (82), with $W(\varphi)$ respectively replaced by the smooth function $U(\varphi)$ depending only on the undifferentiated scalar field φ.

Introducing (A.2) in (66) for $I=5$ and recalling definitions (35)–(52), we obtain

$$\begin{aligned}a_4^{\mathrm{int}}={}&-\frac{1}{6}\tilde{D}_{\mu\alpha}\tilde{D}^{\alpha\beta}\sigma^{\mu\nu}\bigg[(\tilde{U}_1)^{\lambda}\bigg(C_{\lambda}\tilde{D}_{\beta\nu}+\frac{3}{2}C\tilde{F}_{\beta\nu|\lambda}\bigg)\\&-(\tilde{U}_2)^{\lambda}\bigg(\frac{1}{5}\tilde{\mathcal{G}}_{\lambda}\tilde{D}_{\beta\nu}+\frac{3}{2}\tilde{\mathcal{G}}\tilde{F}_{\beta\nu|\lambda}\bigg)\bigg]\\&+\frac{1}{2}(\tilde{U}_3)^{\lambda}(V_{\lambda}S_{\mu}+\eta\mathcal{C}_{\lambda\mu})S^{\mu}\\&-\frac{1}{4}(\tilde{U}_4)^{\lambda}\big[D^{\mu\nu\rho}\big(\tilde{D}_{\nu\alpha}\tilde{D}_{\rho\beta}\sigma^{\alpha\beta}\mathcal{C}_{\lambda\mu}\\&-2\tilde{D}_{\nu\alpha}\tilde{F}_{\rho\beta|\lambda}\sigma^{\alpha\beta}S_{\mu}\big)\\&-F^{\mu\nu\rho|\gamma}\tilde{D}_{\nu\alpha}\tilde{D}_{\rho\beta}S_{\mu}\sigma^{\alpha\beta}\sigma_{\gamma\lambda}\big]+\bar{a}_4^{\mathrm{int}}.\end{aligned} \tag{A.3}$$

In (A.3) $(\tilde{U})^{\lambda}$ are dual to (83), with $W(\varphi)\to U(\varphi)$. In addition, $\mathcal{C}_{\mu\rho}$ is implicitly defined by formula (74) so it is a ghost field of pure ghost number one without definite symmetry/antisymmetry property, $\mathcal{C}^{*\nu\lambda}$ is its associated antifield, defined such that the antibracket $(\mathcal{C}_{\mu\rho},\mathcal{C}^{*\nu\lambda})$ is equal to the 'unit' $\delta_{\mu}^{\nu}\delta_{\rho}^{\lambda}$

$$\mathcal{C}^{*\nu\lambda}\equiv 3S^{*\nu\lambda}+A^{*\nu\lambda}. \tag{A.4}$$

The nonintegrated density $\bar{a}_4^{\mathrm{int}}$ stands for the solution to the homogeneous equation (68) for $I=4$, showing that $\bar{a}_4^{\mathrm{int}}$ can be taken as a nontrivial element of $H(\gamma)$ in pure ghost number equal to four.

At this stage it is useful to decompose $\bar{a}_4^{\mathrm{int}}$ as a sum between two components

$$\bar{a}_4^{\mathrm{int}}=\hat{a}_4^{\mathrm{int}}+\check{a}_4^{\mathrm{int}}, \tag{A.5}$$

where $\hat{a}_4^{\mathrm{int}}$ is the solution to (68) for $I=4$ which is explicitly required by the consistency of a_4^{int} in antighost number three (ensures that (67) possesses solutions for $i=4$ with respect to the terms from (A.3) containing the functions of type U) and $\check{a}_4^{\rm int}$ signifies the part of the solution to (68) for $I=4$ that is independently consistent in antighost number three

$$\delta\check{a}_4^{\rm int} = -\gamma\check{c}_3 + \partial_\mu \check{m}_3^\mu. \tag{A.6}$$

Using definitions (35)–(52) and decomposition (A.5), by direct computation we obtain that

$$\begin{aligned}\delta a_4^{\rm int} = \delta\bigg[&\hat{a}_4^{\rm int} - \frac{1}{2}S^\alpha S_\alpha\bigg((\tilde{U}_3)^{\mu\nu}B^*_{\mu\nu} \\ &+ \frac{1}{3}(\tilde{U}_3)^{\mu\nu\rho}\eta^*_{\mu\nu\rho} + \frac{1}{12}(\tilde{U}_3)^{\mu\nu\rho\lambda}\eta^*_{\mu\nu\rho\lambda} \\ &+ \frac{1}{60}(\tilde{U}_3)^{\mu\nu\rho\lambda\sigma}\eta^*_{\mu\nu\rho\lambda\sigma}\bigg)\bigg] \\ &+ \gamma c_3 + \partial_\mu j_3^\mu + \chi_3,\end{aligned} \tag{A.7}$$

where we use the notation

$$\begin{aligned}c_3 = {}& -\check{c}_3 + \frac{1}{12}(\tilde{U}_1)^{\lambda\sigma}\tilde{D}_{\mu\rho}\sigma^{\mu\nu} \\ &\times\bigg[\tilde{D}^{\rho\alpha}(\phi_{\lambda\sigma}\tilde{D}_{\alpha\nu} - 3C_\lambda\tilde{F}_{\alpha\nu|\sigma}) + \frac{3}{2}C\tilde{F}^{\rho\alpha}{}_{|\lambda}\tilde{F}_{\alpha\nu|\sigma}\bigg] \\ &- \frac{1}{240}(\tilde{U}_2)^{\lambda\sigma}\tilde{D}_{\mu\rho}\sigma^{\mu\nu}\big[\tilde{D}^{\rho\alpha}(\tilde{K}_{\lambda\sigma}\tilde{D}_{\alpha\nu} \\ &- 12\tilde{\mathcal{G}}_\lambda\tilde{F}_{\alpha\nu|\sigma}) - 30\tilde{\mathcal{G}}\tilde{F}^{\rho\alpha}{}_{|\lambda}\tilde{F}_{\alpha\nu|\sigma}\big] \\ &- \frac{1}{12}(\tilde{U}_3)^{\lambda\sigma}\bigg[S^\mu(6V_\lambda\mathcal{C}_{\sigma\mu} - \eta t_{\lambda\sigma|\mu}) \\ &+ \frac{3}{2}\eta\mathcal{C}_{\lambda\rho}\mathcal{C}_{\sigma\mu}\sigma^{\rho\mu}\bigg] \\ &- \frac{1}{2}\bigg((\tilde{U}_3)^{\mu\nu\sigma}B^*_{\mu\nu} + \frac{1}{3}(\tilde{U}_3)^{\mu\nu\rho\sigma}\eta^*_{\mu\nu\rho} \\ &+ \frac{1}{12}(\tilde{U}_3)^{\mu\nu\rho\lambda\sigma}\eta^*_{\mu\nu\rho\lambda}\bigg)S^\alpha\mathcal{C}_{\sigma\alpha} \\ &- \frac{1}{24}(\tilde{U}_4)^{\lambda\sigma}\sigma^{\alpha\beta}\big[D^{\mu\nu\rho}(6\tilde{D}_{\nu\alpha}\tilde{F}_{\rho\beta|\lambda}\mathcal{C}_{\sigma\mu} \\ &+ 3\tilde{F}_{\nu\alpha|\lambda}\tilde{F}_{\rho\beta|\sigma}S_\mu + \tilde{D}_{\nu\alpha}\tilde{D}_{\rho\beta}t_{\lambda\sigma|\mu}) \\ &+ 3F^{\mu\nu\rho}{}_{|\lambda}\tilde{D}_{\nu\alpha}(\tilde{D}_{\rho\beta}\mathcal{C}_{\sigma\mu} - 2\tilde{F}_{\rho\beta|\sigma}S_\mu)\big],\end{aligned} \tag{A.8}$$

$$\begin{aligned}\chi_3 = {}& -\frac{1}{4}\big((\tilde{U}_1)^{\lambda\sigma}C + (\tilde{U}_2)^{\lambda\sigma}\tilde{\mathcal{G}}\big)\sigma^{\mu\nu}\tilde{D}_{\mu\alpha}\tilde{D}^{\alpha\beta}\tilde{R}_{\beta\nu|\lambda\sigma} \\ &+ \frac{1}{6}(\tilde{U}_3)^{\mu\nu}\eta S^\rho D_{\mu\nu\rho} \\ &- \frac{1}{12}(\tilde{U}_4)^{\lambda\sigma}\sigma^{\alpha\beta}\big[-3R^{\mu\nu\rho}_{|\lambda\sigma}\tilde{D}_{\nu\alpha}\tilde{D}_{\rho\beta}S_\mu \\ &+ D^{\mu\nu\rho}\tilde{D}_{\nu\alpha}(\tilde{D}_{\rho\beta}D_{\lambda\sigma\mu} - 6\tilde{R}_{\rho\beta|\lambda\sigma}S_\mu)\big],\end{aligned} \tag{A.9}$$

and j_3^μ are some local currents. In (A.7)–(A.9) $(\tilde{U})^{\mu\nu}$ and $(\tilde{U})^{\mu\nu\rho}$ denote the duals of (84) and (85) with $W(\varphi)\rightarrow U(\varphi)$. In addition, $(\tilde{U})^{\mu\nu\rho\lambda}$ represents the dual of $(U)_\mu = \frac{dU}{d\varphi}H^*_\mu$ and $(\tilde{U})^{\mu\nu\rho\lambda\sigma}$ the dual of $U(\varphi)$. Inspecting (A.7), it follows that the consistency of $a_4^{\rm int}$ in antighost number three, namely the existence of $a_3^{\rm int}$ as solution to (67) for $i=4$, requires the conditions

$$\chi_3 = \gamma\hat{c}_3 + \partial_\mu\hat{j}_3^\mu \tag{A.10}$$

and

$$\begin{aligned}\hat{a}_4^{\rm int} = {}& \frac{1}{2}S^\alpha S_\alpha\bigg((\tilde{U}_3)^{\mu\nu}B^*_{\mu\nu} + \frac{1}{3}(\tilde{U}_3)^{\mu\nu\rho}\eta^*_{\mu\nu\rho} \\ &+ \frac{1}{12}(\tilde{U}_3)^{\mu\nu\rho\lambda}\eta^*_{\mu\nu\rho\lambda} \\ &+ \frac{1}{60}(\tilde{U}_3)^{\mu\nu\rho\lambda\sigma}\eta^*_{\mu\nu\rho\lambda\sigma}\bigg),\end{aligned} \tag{A.11}$$

where we made the notations $\hat{c}_3 = -(a_3^{\rm int} + c_3)$ and $\hat{j}_3^\mu = \overset{(3)}{m}{}^{{\rm int}\,\mu} - j_3^\mu$. Nevertheless, from (A.9) it is obvious that χ_3 is a nontrivial element from $H(\gamma)$ in pure ghost number four, which does not reduce to a full divergence, and therefore (A.10) requires that $\chi_3 = 0$, which further imply that all the functions of type U must be some real constants

$$\begin{aligned}U_1(\varphi) &= u_1, \qquad U_2(\varphi) = u_2, \\ U_3(\varphi) &= u_3, \qquad U_4(\varphi) = u_4.\end{aligned} \tag{A.12}$$

Based on (A.12), it is clear that $a_5^{\rm int}$ given by (A.2) vanishes, and hence we can assume, without loss of nontrivial terms, that

$$a_5^{\rm int} = 0 \tag{A.13}$$

in (A.1).

Appendix B: No-go result for $I=4$ in $a^{\rm int}$

We have seen in Appendix A that we can always take (A.13) in (A.1). Consequently, the first-order deformation of the solution to the master equation in the interacting case stops at antighost number four

$$a^{\rm int} = a_0^{\rm int} + a_1^{\rm int} + a_2^{\rm int} + a_3^{\rm int} + a_4^{\rm int}, \tag{B.1}$$

where the components on the right-hand side of (B.1) are subject to (68) and (66)–(67) for $I=4$.

The piece $a_4^{\rm int}$ as solution to (68) for $I=4$ has the general form expressed by (75) for $I=4$, with α_4 from $H_4^{\rm inv}(\delta|d)$. According to (81) at antighost number four, it follows that $H_4^{\rm inv}(\delta|d)$ is spanned by some representatives involving only BF generators. Since $a_4^{\rm int}$ should again mix the BF and the (2, 1) tensor field sectors, it follows that one should retain from the basis elements $e^4(\eta^{\bar{\Upsilon}})$ only the objects containing at least one ghost from the (2, 1) tensor field sector, namely $D_{\mu\nu\rho}$ or S_μ. The general solution to (68) for $I = 4$ reads

$$a_4^{\rm int} = \frac{1}{2}\tilde{\eta}^*\left(q_1 S_\mu S^\mu + \frac{q_2}{3}\sigma^{\mu\nu}\tilde{D}_{\mu\alpha}\tilde{D}^{\alpha\beta}\tilde{D}_{\beta\nu}\eta\right) + (\tilde{U}_5)^\mu\eta\tilde{D}_{\mu\nu}S^\nu + \left((\tilde{U}_6)^\mu C + (\tilde{U}_7)^\mu\tilde{\mathcal{G}}\right)S_\mu - \frac{1}{4}(U_8)_{\mu\nu\rho\lambda}\tilde{D}^{\mu\alpha}\tilde{D}^{\nu\beta}\tilde{D}^{\rho\gamma}\tilde{D}^{\lambda\delta}\sigma_{\alpha(\gamma}\sigma_{\delta)\beta} - \frac{1}{2}(\tilde{U}_9)^\mu D_{\mu\nu\rho}\tilde{D}^{\nu\alpha}\tilde{D}^{\rho\beta}\eta\sigma_{\alpha\beta}, \tag{B.2}$$

where each element generically denoted by $(\tilde{U})^\mu$ is the Hodge dual of an object similar to (83), but with W replaced by the arbitrary, smooth function U, depending on the undifferentiated scalar field, $(U_8)_{\mu\nu\rho\lambda}$ reads as in (83) with $W(\varphi) \to U_8(\varphi)$, and $q_{1,2}$ are two arbitrary, real constants.

Introducing (B.2) in (66) for $I = 4$ and using definitions (35)–(52), we determine the component of antighost number three from $a^{\rm int}$ in the form

$$a_3^{\rm int} = \frac{1}{2}q_1\tilde{\eta}^{*\mu}S^\nu\mathcal{C}_{\mu\nu} + \frac{1}{6}q_2\tilde{\eta}^{*\lambda}\sigma^{\mu\nu}\tilde{D}_{\mu\alpha}\tilde{D}^{\alpha\beta}\left(\tilde{D}_{\beta\nu}V_\lambda + \frac{3}{2}\tilde{F}_{\beta\nu|\lambda}\eta\right) + \frac{1}{2}(\tilde{U}_5)^{\mu\nu}\left[(2V_\mu\tilde{D}_{\nu\rho} - \eta\tilde{F}_{\mu\rho|\nu})S^\rho + \sigma^{\rho\lambda}\eta\tilde{D}_{\mu\rho}\mathcal{C}_{\nu\lambda}\right] - \frac{1}{2}(\tilde{U}_6)^{\mu\nu}(A_{\mu\nu}C - 2S_\mu C_\nu) - \frac{1}{2}(\tilde{U}_7)^{\mu\nu}\left(A_{\mu\nu}\tilde{\mathcal{G}} + \frac{2}{5}S_\mu\tilde{\mathcal{G}}_\nu\right) - \frac{1}{2}(\tilde{U}_9)^{\mu\nu}\sigma_{\alpha\beta}\left[D_{\mu\lambda\rho}\tilde{D}^{\lambda\alpha}\left(\tilde{D}^{\rho\beta}V_\nu + \tilde{F}^{\rho\beta}{}_{|\nu}\eta\right) + \frac{1}{2}F_{\mu\lambda\rho|\nu}\tilde{D}^{\lambda\alpha}\tilde{D}^{\rho\beta}\eta\right] - \frac{1}{2}(\tilde{U}_8)^{\mu\tau}\varepsilon_{\mu\nu\rho\lambda\sigma}\tilde{D}^{\nu\alpha}\tilde{D}^{\rho\beta}\tilde{D}^{\lambda\gamma}\tilde{F}^{\sigma\delta}{}_{|\tau}\sigma_{\alpha(\gamma}\sigma_{\delta)\beta} + \bar{a}_3^{\rm int}, \tag{B.3}$$

where each $(\tilde{U})^{\mu\nu}$ is the Hodge dual of an object of type (84), with W replaced by the corresponding function of type U. Here, $\bar{a}_3^{\rm int}$ is the general solution to the homogeneous equation (68) for $I = 3$, showing that $\bar{a}_3^{\rm int}$ is a nontrivial object from $H(\gamma)$ in pure ghost number three.

At this point we decompose $\bar{a}_3^{\rm int}$ in a manner similar to (A.5)

$$\bar{a}_3^{\rm int} = \hat{a}_3^{\rm int} + \check{a}_3^{\rm int}, \tag{B.4}$$

where $\hat{a}_3^{\rm int}$ is the solution to (68) for $I = 3$ that ensures the consistency of $a_3^{\rm int}$ in antighost number two, namely the existence of $a_2^{\rm int}$ as solution to (67) for $i = 3$ with respect to the terms from $a_3^{\rm int}$ containing the functions of type U or the constants q_1 or q_2, while $\check{a}_3^{\rm int}$ is the solution to (68) for $I = 3$ which is independently consistent in antighost number two

$$\delta\check{a}_3^{\rm int} = -\gamma\check{c}_2 + \partial_\mu\check{m}_2^\mu. \tag{B.5}$$

Based on definitions (35)–(52) and taking into account decomposition (B.4), we get by direct computation

$$\delta a_3^{\rm int} = \delta\Bigg[\hat{a}_3^{\rm int} - \frac{q_2}{6}\tilde{\eta}^{*\lambda\sigma}\sigma^{\mu\nu}\tilde{D}_{\mu\alpha}\tilde{D}^{\alpha\beta}\tilde{D}_{\beta\nu}B^*_{\lambda\sigma} - \left((\tilde{U}_5)^{\mu\nu\alpha}B^*_{\mu\nu} + \frac{1}{3}(\tilde{U}_5)^{\mu\nu\rho\alpha}\eta^*_{\mu\nu\rho} + \frac{1}{12}(\tilde{U}_5)^{\mu\nu\rho\lambda\alpha}\eta^*_{\mu\nu\rho\lambda}\right)\tilde{D}_{\alpha\beta}S^\beta + \frac{1}{2}\sigma_{\alpha\beta}D_{\mu\nu\rho}\tilde{D}^{\nu\alpha}\tilde{D}^{\rho\beta}\left((\tilde{U}_9)^{\mu\lambda\sigma}B^*_{\lambda\sigma} - \frac{1}{3}(\tilde{U}_9)^{\mu\lambda\sigma\gamma}\eta^*_{\lambda\sigma\gamma} + \frac{1}{12}(\tilde{U}_9)^{\mu\lambda\sigma\gamma\delta}\eta^*_{\lambda\sigma\gamma\delta}\right)\Bigg] + \gamma c_2 + \partial_\lambda j_2^\lambda + \chi_2, \tag{B.6}$$

where

$$c_2 = -\check{c}_2 + \frac{q_1}{12}\tilde{\eta}^{*\mu\nu}\left(S^\rho t_{\mu\nu|\rho} - \frac{3}{2}\sigma^{\rho\lambda}\mathcal{C}_{\mu\rho}\mathcal{C}_{\nu\lambda}\right) + \frac{q_2}{4}\tilde{\eta}^{*\lambda\sigma}\sigma^{\mu\nu}\tilde{D}_{\mu\alpha}\left(\tilde{D}^{\alpha\beta}V_\lambda + \frac{1}{2}\tilde{F}^{\alpha\beta}{}_{|\lambda}\eta\right)\tilde{F}_{\beta\nu|\sigma} + \frac{1}{2}(\tilde{U}_5)^{\mu\nu\rho}\left[V_\mu\left(\tilde{F}_{\nu\lambda|\rho}S^\lambda - \tilde{D}_{\nu\lambda}\mathcal{C}^\lambda_\rho\right) + \frac{1}{2}\eta\left(\tilde{F}_{\mu\lambda|\nu}\mathcal{C}_\rho{}^\lambda + \frac{1}{3}\tilde{D}_\mu{}^\alpha t_{\nu\rho|\alpha}\right)\right] + \frac{1}{2}\left((\tilde{U}_5)^{\mu\nu\lambda\sigma}B^*_{\mu\nu} + \frac{1}{3}(\tilde{U}_5)^{\mu\nu\rho\lambda\sigma}\eta^*_{\mu\nu\rho}\right) \times \left(\tilde{F}_{\lambda\alpha|\sigma}S^\alpha - \tilde{D}_{\lambda\alpha}\mathcal{C}^\alpha_\sigma\right) + \frac{1}{2}(\tilde{U}_6)^{\mu\nu\rho}(A_{\mu\nu}C_\rho + S_\mu\phi_{\nu\rho}) - \frac{1}{10}(\tilde{U}_7)^{\mu\nu\rho}\left(A_{\mu\nu}\tilde{\mathcal{G}}_\rho + \frac{1}{4}S_\mu\tilde{K}_{\nu\rho}\right) + \frac{1}{8}(\tilde{U}_8)^{\mu\varepsilon\pi}\varepsilon_{\mu\nu\rho\lambda\sigma}\tilde{D}^{\nu\alpha}\left(\tilde{D}^{\rho\beta}\tilde{F}^{\lambda\gamma}{}_{|\varepsilon} + 2\tilde{F}^{\rho\beta}{}_{|\varepsilon}\tilde{D}^{\lambda\gamma}\right)\tilde{F}^{\sigma\delta}{}_{|\pi}\sigma_{\alpha(\gamma}\sigma_{\delta)\beta} - \frac{1}{8}(\tilde{U}_9)^{\mu\lambda\sigma}\sigma_{\alpha\beta}\left[D_{\mu\nu\rho}\left(4\tilde{D}^{\nu\alpha}\tilde{F}^{\rho\beta}{}_{|\sigma}V_\lambda + \tilde{F}^{\nu\alpha}{}_{|\lambda}\tilde{F}^{\rho\beta}{}_{|\sigma}\eta\right) + 2F_{\mu\nu\rho|\sigma}\tilde{D}^{\nu\alpha}\left(\tilde{D}^{\rho\beta}V_\lambda + \tilde{F}^{\rho\beta}{}_{|\lambda}\eta\right)\right]$$

$$
\begin{aligned}
&-\frac{1}{4}(\tilde{U}_9)^{\mu\lambda\sigma\gamma}\sigma_{\alpha\beta}\big(2D_{\mu\nu\rho}\tilde{F}^{\nu\alpha}{}_{|\gamma}\\
&-F_{\mu\nu\rho|\gamma}\tilde{D}^{\nu\alpha}\big)\tilde{D}^{\rho\beta}B^{*}_{\lambda\sigma}\\
&+\frac{1}{12}(\tilde{U}_9)^{\mu\lambda\sigma\gamma\delta}\sigma_{\alpha\beta}\big(2D_{\mu\nu\rho}\tilde{F}^{\nu\alpha}{}_{|\delta}\\
&-F_{\mu\nu\rho|\delta}\tilde{D}^{\nu\alpha}\big)\tilde{D}^{\rho\beta}\eta^{*}_{\lambda\sigma\gamma},
\end{aligned}
\tag{B.7}
$$

$$
\begin{aligned}
\chi_2 = {}&\frac{q_1}{6}\tilde{\eta}^{*\mu\nu}S^{\rho}D_{\mu\nu\rho}\\
&+\frac{1}{6}(\tilde{U}_5)^{\mu\nu\rho}\eta\big(\tilde{D}^{\alpha}_{\mu}D_{\nu\rho\alpha}-3\tilde{R}_{\mu\lambda|\nu\rho}S^{\lambda}\big)\\
&+\frac{q_2}{6}\sigma^{\mu\nu}\tilde{D}_{\mu\alpha}\tilde{D}^{\alpha\beta}\Big[\big(\partial_{\sigma}\tilde{B}^{*\lambda\rho\sigma}\big)\tilde{D}_{\beta\nu}B^{*}_{\lambda\rho}\\
&+\frac{3}{2}\tilde{\eta}^{*\lambda\rho}\tilde{R}_{\beta\nu|\lambda\rho}\eta\Big]\\
&+\frac{1}{6}(\tilde{U}_6)^{\mu\nu\rho}D_{\mu\nu\rho}C+\frac{1}{6}(\tilde{U}_7)^{\mu\nu\rho}D_{\mu\nu\rho}\tilde{\mathcal{G}}\\
&-\frac{1}{2}(\tilde{U}_8)^{\mu\varepsilon\pi}\varepsilon_{\mu\nu\rho\lambda\sigma}\sigma_{\alpha(\gamma}\sigma_{\delta)\beta}\tilde{D}^{\nu\alpha}\tilde{D}^{\rho\beta}\tilde{D}^{\lambda\gamma}\tilde{R}^{\sigma\delta}{}_{|\varepsilon\pi}\\
&+\frac{1}{4}(\tilde{U}_9)^{\mu\lambda\sigma}\sigma_{\alpha\beta}\big(2D_{\mu\nu\rho}\tilde{R}^{\nu\alpha}{}_{|\lambda\sigma}\\
&-R_{\mu\nu\rho|\lambda\sigma}\tilde{D}^{\nu\alpha}\big)\tilde{D}^{\rho\beta}\eta,
\end{aligned}
\tag{B.8}
$$

and j_2^{μ} are some local currents. Reprising an argument similar to that employed in Appendix A between (A.10) and (A.13), we find that the existence of a_2^{int} as solution to (67) for $i=3$ finally implies that χ_2 expressed by (B.8) must vanish. This is further equivalent to the fact that all the functions of type U must be some real constants and both constants $q_{1,2}$ must vanish

$$
U_5(\varphi)=u_5, \qquad U_6(\varphi)=u_6, \qquad U_7(\varphi)=u_7, \tag{B.9}
$$

$$
U_8(\varphi)=u_8, \qquad U_9(\varphi)=u_9, \qquad q_1=0=q_2. \tag{B.10}
$$

Inserting (B.9) and (B.9) in (B.2), we conclude that we can safely take

$$
a_4^{\text{int}}=0 \tag{B.11}
$$

in (B.1).

Appendix C: No-go result for $I=3$ in a^{int}

We have seen in the previous two Appendixes A and B that we can always take (A.13) and (B.11) in (A.1). Consequently, the first-order deformation of the solution to the master equation in the interacting case stops at antighost number three

$$
a^{\text{int}}=a_0^{\text{int}}+a_1^{\text{int}}+a_2^{\text{int}}+a_3^{\text{int}}, \tag{C.1}
$$

where the components on the right-hand side of (C.1) are subject to (68) and (66)–(67) for $I=3$.

The piece a_3^{int} as solution to (68) for $I=3$ has the general form expressed by (75) for $I=3$, with α_3 from $H_3^{\text{inv}}(\delta|d)$. Looking at formula (76) and also at relation (81) in antighost number three and requiring that a_3^{int} mixes BRST generators from the BF and (2, 1) sectors, we find that the most general solution to (68) for $I=3$ reads[3]

$$
\begin{aligned}
a_3^{\text{int}}={}&\tilde{\eta}^{*\mu}\Big(q_3\eta S_{\mu}+q_4S^{\nu}\tilde{D}_{\mu\nu}\\
&-\frac{1}{2}q_5\sigma_{\alpha\beta}D_{\mu\nu\rho}\tilde{D}^{\nu\alpha}\tilde{D}^{\rho\beta}\Big)+q_6S^{*\mu}\eta S_{\mu}\\
&+\frac{1}{6}\sigma^{\mu\nu}\big(q_7C^{*}+q_8\tilde{\mathcal{G}}^{*}\big)\tilde{D}_{\mu\alpha}\tilde{D}^{\alpha\beta}\tilde{D}_{\beta\nu}\\
&+(\tilde{U}_{10})^{\mu\nu}\tilde{D}_{\mu\nu}\tilde{\mathcal{G}}+(\tilde{U}_{11})^{\mu\nu}\tilde{D}_{\mu\nu}C\\
&+\frac{1}{2}(\tilde{U}_{12})^{\mu\nu}\sigma^{\alpha\beta}\eta\tilde{D}_{\mu\alpha}\tilde{D}_{\nu\beta},
\end{aligned}
\tag{C.2}
$$

where any object denoted by q represents an arbitrary, real constant. Inserting (C.2) in (66) for $I=3$ and using definitions (35)–(52), we can write

$$
\begin{aligned}
a_2^{\text{int}}={}&-q_3\tilde{\eta}^{*\mu\nu}\Big(V_{\mu}S_{\nu}+\frac{1}{2}\eta A_{\mu\nu}\Big)\\
&+\frac{q_4}{2}\tilde{\eta}^{*\mu\nu}\big(\mathcal{C}_{\mu\rho}\tilde{D}^{\rho}_{\nu}+S^{\rho}\tilde{F}_{\rho\mu|\nu}\big)\\
&-\frac{q_5}{4}\tilde{\eta}^{*\mu\varepsilon}\sigma_{\alpha\beta}\big(2D_{\mu\nu\rho}\tilde{F}^{\nu\alpha}{}_{|\varepsilon}-F_{\mu\nu\rho|\varepsilon}\tilde{D}^{\nu\alpha}\big)\tilde{D}^{\rho\beta}\\
&-q_6\mathcal{C}^{*\mu\nu}\big(2V_{\mu}S_{\nu}+\eta\mathcal{C}_{\mu\nu}\big)\\
&+\frac{1}{4}\sigma^{\mu\nu}\big(q_7C^{*\lambda}-q_8\tilde{\mathcal{G}}^{*\lambda}\big)\tilde{D}_{\mu\alpha}\tilde{D}^{\alpha\beta}\tilde{F}_{\beta\nu|\lambda}\\
&-\frac{1}{2}(\tilde{U}_{10})^{\mu\nu\rho}\Big(\tilde{F}_{\mu\nu|\rho}\tilde{\mathcal{G}}+\frac{2}{5}\tilde{D}_{\mu\nu}\tilde{\mathcal{G}}_{\rho}\Big)\\
&+(\tilde{U}_{11})^{\mu\nu\rho}\Big(\tilde{D}_{\mu\nu}C_{\rho}-\frac{1}{2}\tilde{F}_{\mu\nu|\rho}C\Big)\\
&+\frac{1}{2}(\tilde{U}_{12})^{\mu\nu\rho}\sigma^{\alpha\beta}\big(V_{\mu}\tilde{D}_{\nu\alpha}\\
&+\eta\tilde{F}_{\alpha\mu|\nu}\big)\tilde{D}_{\rho\beta}+\bar{a}_2^{\text{int}}.
\end{aligned}
\tag{C.3}
$$

The component $\bar{a}_2^{\text{int}}$ represents the solution to the homogeneous equation in antighost number two (68) for $I=2$, so $\bar{a}_2^{\text{int}}$ is a nontrivial element from $H(\gamma)$ of pure ghost number two and antighost number two. It is useful to decompose $\bar{a}_2^{\text{int}}$ as a sum between two terms

$$
\bar{a}_2^{\text{int}}=\hat{a}_2^{\text{int}}+\check{a}_2^{\text{int}}, \tag{C.4}
$$

[3] In principle, one can add to a_3^{int} the term $(M_1)_{\mu\nu\rho}\tilde{D}^{\mu\nu}S^{\rho}$, where $(M_1)_{\mu\nu\rho}$ reads as in (84), with $W(\varphi)\rightarrow M_1(\varphi)$. It is possible to show that such a term outputs only trivial deformations.

with $\hat{a}_2^{\mathrm{int}}$ the solution to (68) for $I=2$ that ensures the consistency of a_2^{int} in antighost number one, namely the existence of a_1^{int} as solution to (67) for $i=2$ with respect to the terms from a_2^{int} containing the functions of type U or the constants denoted by q, and $\check{a}_2^{\mathrm{int}}$ the solution to (68) for $I=2$ that is independently consistent in antighost number one

$$\delta \check{a}_2^{\mathrm{int}} = -\gamma \check{c}_1 + \partial_\mu \check{m}_1^\mu. \tag{C.5}$$

Using definitions (35)–(49) and decomposition (C.4), by direct computation we obtain that

$$\begin{aligned}\delta a_2^{\mathrm{int}} = \delta\Big[&\hat{a}_2^{\mathrm{int}} - \frac{1}{2}\Big((\tilde{U}_{12})^{\mu\nu\lambda\sigma} B^*_{\mu\nu} \\ &+ \frac{1}{3}(\tilde{U}_{12})^{\mu\nu\rho\lambda\sigma}\eta^*_{\mu\nu\rho}\Big)\sigma^{\alpha\beta}\tilde{D}_{\lambda\alpha}\tilde{D}_{\sigma\beta}\Big] \\ &+ \gamma c_1 + \partial_\lambda j_1^\lambda + \chi_1,\end{aligned} \tag{C.6}$$

where we used the notations

$$\begin{aligned}c_1 = -\check{c}_1 - \frac{1}{2}\tilde{B}^{*\mu\nu\rho}\Big[&q_3 V_\mu A_{\nu\rho} \\ &- \frac{q_4}{6}\sigma^{\alpha\beta}(3\mathcal{C}_{\mu\alpha}\tilde{F}_{\nu\beta|\rho} + t_{\mu\nu|\alpha}\tilde{D}_{\rho\beta}) \\ &+ \frac{q_5}{4}\sigma_{\lambda\sigma}\big(D_{\mu\alpha\beta}\tilde{F}^{\alpha\lambda}{}_{|\nu} - 2F_{\mu\alpha\beta|\nu}\tilde{D}^{\alpha\lambda}\big)\tilde{F}^{\beta\sigma}{}_{|\rho}\Big] \\ &+ q_6 t^{*\mu\nu|\rho}(6V_\mu \mathcal{C}_{\nu\rho} - \eta t_{\mu\nu|\rho}) \\ &+ \frac{1}{2}(\tilde{U}_{11})^{\mu\nu\rho\lambda}(\tilde{F}_{\mu\nu|\rho}C_\lambda + \tilde{D}_{\mu\nu}\phi_{\rho\lambda}) \\ &- \frac{1}{4}\sigma^{\mu\nu}\Big(q_7\phi^{*\lambda\rho} - \frac{q_8}{2}\tilde{K}^{*\lambda\rho}\Big)\tilde{D}_{\mu\alpha}\tilde{F}^{\alpha\beta}{}_{|\lambda}\tilde{F}_{\beta\nu|\rho} \\ &- \frac{1}{10}(\tilde{U}_{10})^{\mu\nu\rho\lambda}\Big(\tilde{F}_{\mu\nu|\rho}\tilde{\mathcal{G}}_\lambda + \frac{1}{4}\tilde{D}_{\mu\nu}\tilde{K}_{\rho\lambda}\Big) \\ &+ \frac{1}{2}(\tilde{U}_{12})^{\mu\nu\rho\lambda}\sigma^{\alpha\beta}\Big(V_\mu\tilde{D}_{\nu\alpha}\tilde{F}_{\rho\beta|\lambda} - \frac{1}{4}\eta\tilde{F}_{\mu\alpha|\nu}\tilde{F}_{\rho\beta|\lambda}\Big) \\ &+ \frac{1}{2}(\tilde{U}_{12})^{\mu\nu\rho\lambda\sigma}\sigma^{\alpha\beta}B^*_{\mu\nu}\tilde{D}_{\sigma\alpha}\tilde{F}_{\rho\beta|\lambda},\end{aligned} \tag{C.7}$$

$$\begin{aligned}\chi_1 = \frac{1}{6}\tilde{B}^{*\mu\nu\rho}\Big[&q_3(\eta D_{\mu\nu\rho} + 3S_\rho\partial_{[\mu}V_{\nu]}) \\ &- q_4\sigma^{\alpha\beta}(D_{\mu\nu\alpha}\tilde{D}_{\rho\beta} + 3\tilde{R}_{\mu\alpha|\nu\rho}S_\beta) \\ &+ \frac{3q_5}{2}\sigma_{\lambda\sigma}\big(R_{\mu\alpha\beta|\nu\rho}\tilde{D}^{\alpha\lambda} - 2D_{\mu\alpha\beta}\tilde{R}^{\alpha\lambda}{}_{|\nu\rho}\big)\tilde{D}^{\beta\sigma}\Big] \\ &- 6q_6 t^{*\mu\nu|\rho}(\partial_{[\mu}V_{\nu]})S_\rho \\ &+ \frac{1}{2}\sigma^{\mu\nu}\Big(q_7\phi^{*\rho\lambda} - \frac{q_8}{2}\tilde{K}^{*\rho\lambda}\Big)\tilde{D}_{\mu\alpha}\tilde{D}^{\alpha\beta}\tilde{R}_{\beta\nu|\rho\lambda} \\ &- \frac{1}{2}(\tilde{U}_{10})^{\mu\nu\rho\lambda}\tilde{R}_{\mu\nu|\rho\lambda}\tilde{\mathcal{G}} \\ &- \frac{1}{2}(\tilde{U}_{11})^{\mu\nu\rho\lambda}\tilde{R}_{\mu\nu|\rho\lambda}C \\ &- \frac{1}{2}(\tilde{U}_{12})^{\mu\nu\rho\lambda}\sigma^{\alpha\beta}\eta\tilde{D}_{\mu\alpha}\tilde{R}_{\nu\beta|\rho\lambda},\end{aligned} \tag{C.8}$$

and j_1^μ are some local currents. It is easy to see that χ_1 given in (C.8) is a nontrivial object from $H(\gamma)$ in pure ghost number two, which obviously does not reduce to a full divergence. Then, since (C.6) requires that it is γ-exact modulo d, it must vanish, which further implies that all the functions of type $U(\varphi)$ are some real constants and all the constants denoted by q vanish

$$\begin{aligned}&U_{10}(\varphi) = u_{10}, \qquad U_{11}(\varphi) = u_{11}, \\ &U_{12}(\varphi) = u_{12}, \\ &q_3 = q_4 = q_5 = q_6 = q_7 = q_8 = 0.\end{aligned} \tag{C.9}$$

Inserting conditions (C.9) and (C.10) into (C.2), we conclude that we conclude that we can safely take

$$a_3^{\mathrm{int}} = 0 \tag{C.10}$$

in (C.1).

Appendix D: No-go result for $I=0$ in a^{int}

The solution to the 'homogeneous' equation (119) can be represented as

$$\bar{a}_0^{\mathrm{int}} = \bar{a}_0'^{\mathrm{int}} + \bar{a}_0''^{\mathrm{int}}, \tag{D.1}$$

where

$$\gamma \bar{a}_0'^{\mathrm{int}} = 0, \tag{D.2}$$

$$\gamma \bar{a}_0''^{\mathrm{int}} = \partial_\mu \bar{m}_0^\mu \tag{D.3}$$

and $\bar{m}_0^\mu$ is a nonvanishing, local current.

According to the general result expressed by (75) in both antighost and pure ghost numbers equal to zero, (D.2) implies

$$\bar{a}_0'^{\mathrm{int}} = \bar{a}_0'^{\mathrm{int}}([F_{\bar{A}}]), \tag{D.4}$$

where $F_{\bar{A}}$ are listed in (75). Solution (D.4) is assumed to provide a cross-coupling Lagrangian. Therefore, since $R_{\mu\nu\rho|\alpha\beta}$ is the most general gauge-invariant quantity depending on the field $t_{\mu\nu|\alpha}$, it follows that each interaction vertex from $\bar{a}_0'^{\mathrm{int}}$ is required to be at least linear in $R_{\mu\nu\rho|\alpha\beta}$ and to depend at least on a BF field. But $R_{\mu\nu\rho|\alpha\beta}$ contains two spacetime derivatives, so the emerging interacting field equations would exhibit at least two spacetime derivatives acting on the BF field(s) from the interaction vertices. Nevertheless, this contradicts the general assumption on the preservation of the differential order of each field equation with respect to the free theory (see assumption (ii) from the beginning of Sect. 4), so we must set

$$\bar{a}_0^{\prime\mathrm{int}} = 0. \tag{D.5}$$

Next, we solve (D.3). In view of this, we decompose $\bar{a}_0^{\prime\prime\mathrm{int}}$ with respect to the number of derivatives acting on the fields as

$$\bar{a}_0^{\prime\prime\mathrm{int}} = \overset{(0)}{\pi} + \overset{(1)}{\pi} + \overset{(2)}{\pi}, \tag{D.6}$$

where each $\overset{(i)}{\pi}$ contains precisely i spacetime derivatives. Of course, each $\overset{(i)}{\pi}$ is required to mix the BF and (2, 1) field sectors in order to produce cross-interactions. In agreement with (D.6), equation (D.3) is equivalent to

$$\gamma \overset{(0)}{\pi} = \partial_\mu \overset{(0)}{m}{}_0^\mu, \tag{D.7}$$

$$\gamma \overset{(1)}{\pi} = \partial_\mu \overset{(1)}{m}{}_0^\mu, \tag{D.8}$$

$$\gamma \overset{(2)}{\pi} = \partial_\mu \overset{(2)}{m}{}_0^\mu. \tag{D.9}$$

Using definitions (45)–(47) and an integration by parts it is possible to show that

$$\begin{aligned}\gamma \overset{(0)}{\pi} = {}& \partial_\mu \overset{(0)}{m}{}_0^\mu - \left(\partial_\mu \frac{\partial \overset{(0)}{\pi}}{\partial t_{\mu(\nu|\alpha)}}\right) S_{\nu\alpha} \\ &+ \left(2\partial_\mu \frac{\partial \overset{(0)}{\pi}}{\partial t_{\alpha\beta|\mu}} - \partial_\mu \frac{\partial \overset{(0)}{\pi}}{\partial t_{\mu[\alpha|\beta]}}\right) A_{\alpha\beta} \\ &+ \left(\partial_{[\mu} \frac{\partial \overset{(0)}{\pi}}{\partial H^{\nu]}}\right) C^{\mu\nu} - \left(\partial_\mu \frac{\partial \overset{(0)}{\pi}}{\partial V_\mu}\right)\eta \\ &+ \left(\partial_{[\mu} \frac{\partial \overset{(0)}{\pi}}{\partial B^{\nu\rho]}}\right)\eta^{\mu\nu\rho} \\ &- 2\left(\partial_\mu \frac{\partial \overset{(0)}{\pi}}{\partial \phi_{\mu\nu}}\right) C_\nu + \left(\partial_{[\mu} \frac{\partial \overset{(0)}{\pi}}{\partial K^{\nu\rho\lambda]}}\right)\mathcal{G}^{\mu\nu\rho\lambda}.\end{aligned} \tag{D.10}$$

From (D.10) we observe that $\overset{(0)}{\pi}$ is solution to (D.7) if and only if the following conditions are satisfied simultaneously

$$\partial_\mu \frac{\partial \overset{(0)}{\pi}}{\partial t_{\mu(\nu|\alpha)}} = 0, \qquad \partial_\mu \frac{\partial \overset{(0)}{\pi}}{\partial t_{\alpha\beta|\mu}} = 0, \qquad \partial_{[\mu} \frac{\partial \overset{(0)}{\pi}}{\partial H^{\nu]}} = 0, \tag{D.11}$$

$$\partial_\mu \frac{\partial \overset{(0)}{\pi}}{\partial V_\mu} = 0, \qquad \partial_{[\mu} \frac{\partial \overset{(0)}{\pi}}{\partial B^{\nu\rho]}} = 0, \qquad \partial_\mu \frac{\partial \overset{(0)}{\pi}}{\partial \phi_{\mu\nu}} = 0, \qquad \partial_{[\mu} \frac{\partial \overset{(0)}{\pi}}{\partial K^{\nu\rho\lambda]}} = 0. \tag{D.12}$$

Because $\overset{(0)}{\pi}$ is derivative-free, the solutions to (D.11)–(D.12) read

$$\frac{\partial \overset{(0)}{\pi}}{\partial t_{\mu\nu|\alpha}} = \tau^{\mu\nu|\alpha}, \qquad \frac{\partial \overset{(0)}{\pi}}{\partial H^\mu} = h_\mu, \qquad \frac{\partial \overset{(0)}{\pi}}{\partial V_\mu} = v^\mu, \tag{D.13}$$

$$\frac{\partial \overset{(0)}{\pi}}{\partial B^{\mu\nu}} = b_{\mu\nu}, \qquad \frac{\partial \overset{(0)}{\pi}}{\partial \phi_{\mu\nu}} = f_{\mu\nu}, \qquad \frac{\partial \overset{(0)}{\pi}}{\partial K^{\mu\nu\rho}} = k_{\mu\nu\rho}, \tag{D.14}$$

where $\tau^{\mu\nu|\alpha}$, h_μ, v^μ, $b_{\mu\nu}$, $f_{\mu\nu}$, and $k_{\mu\nu\rho}$ are some real, constant tensors. In addition, $\tau^{\mu\nu|\alpha}$ display the same mixed symmetry properties like the tensor field $t^{\mu\nu|\alpha}$ and $b_{\mu\nu}$, $f_{\mu\nu}$, and $k_{\mu\nu\rho}$ are completely antisymmetric. Because there are no such constant tensors in $D = 5$, we conclude that (D.11)–(D.12) possess only the trivial solution, which further implies that

$$\overset{(0)}{\pi} = 0. \tag{D.15}$$

Related to (D.8), we use again definitions (45)–(47) and integrate twice by parts, obtaining

$$\begin{aligned}\gamma \overset{(1)}{\pi} = {}& \partial_\mu \overset{(1)}{m}{}_0^\mu - \left(\partial_\mu \frac{\delta \overset{(1)}{\pi}}{\delta t_{\mu(\alpha|\beta)}}\right) S_{\alpha\beta} \\ &- \left(\partial_\mu \frac{\delta \overset{(1)}{\pi}}{\delta t_{\mu[\alpha|\beta]}} - 2\partial_\mu \frac{\delta \overset{(1)}{\pi}}{\delta t_{\alpha\beta|\mu}}\right) A_{\alpha\beta} \\ &+ \left(\partial_{[\mu} \frac{\delta \overset{(1)}{\pi}}{\delta H^{\nu]}}\right) C^{\mu\nu} - \left(\partial_\mu \frac{\delta \overset{(1)}{\pi}}{\delta V_\mu}\right)\eta \\ &+ \left(\partial_{[\mu} \frac{\delta \overset{(1)}{\pi}}{\delta B^{\nu\rho]}}\right)\eta^{\mu\nu\rho} \\ &- 2\left(\partial_\mu \frac{\delta \overset{(1)}{\pi}}{\delta \phi_{\mu\nu}}\right) C_\nu + \left(\partial_{[\mu} \frac{\delta \overset{(1)}{\pi}}{\delta K^{\nu\rho\lambda]}}\right)\mathcal{G}^{\mu\nu\rho\lambda}.\end{aligned} \tag{D.16}$$

Inspecting (D.16), we observe that $\overset{(1)}{\pi}$ satisfies equation (D.8) if and only if the following relations take place simultaneously

$$\partial_\mu \frac{\delta \overset{(1)}{\pi}}{\delta t_{\mu(\alpha|\beta)}} = 0, \qquad \partial_\mu \frac{\delta \overset{(1)}{\pi}}{\delta t_{\alpha\beta|\mu}} = 0, \qquad \partial_{[\mu} \frac{\delta \overset{(1)}{\pi}}{\delta H^{\nu]}} = 0, \tag{D.17}$$

$$\partial_\mu \frac{\delta \overset{(1)}{\pi}}{\delta V_\mu} = 0, \qquad \partial_{[\mu} \frac{\delta \overset{(1)}{\pi}}{\delta B^{\nu\rho]}} = 0, \qquad \partial_\mu \frac{\delta \overset{(1)}{\pi}}{\delta \phi_{\mu\nu}} = 0, \qquad \partial_{[\mu} \frac{\delta \overset{(1)}{\pi}}{\delta K^{\nu\rho\lambda]}} = 0. \tag{D.18}$$

The solutions to (D.17)–(D.18) are expressed by

$$\frac{\delta \overset{(1)}{\pi}}{\delta t_{\mu(\alpha|\beta)}} = \partial_\nu s^{\mu\nu\alpha\beta}, \qquad \frac{\delta \overset{(1)}{\pi}}{\delta t_{\alpha\beta|\mu}} = \partial_\nu \tau^{\alpha\beta\mu\nu}, \tag{D.19}$$

$$\frac{\delta \overset{(1)}{\pi}}{\delta H^\mu} = \partial_\mu h, \qquad \frac{\delta \overset{(1)}{\pi}}{\delta V_\mu} = \partial_\nu v^{\mu\nu}, \tag{D.20}$$

$$\frac{\delta \overset{(1)}{\pi}}{\delta B^{\mu\nu}} = \partial_{[\mu} b_{\nu]}, \qquad \frac{\delta \overset{(1)}{\pi}}{\delta \phi_{\mu\nu}} = \partial_\rho f^{\mu\nu\rho}, \qquad \frac{\delta \overset{(1)}{\pi}}{\delta K^{\mu\nu\rho}} = \partial_{[\mu} k_{\nu\rho]}, \tag{D.21}$$

where the quantities $s^{\mu\nu\alpha\beta}$, $\tau^{\alpha\beta\mu\nu}$, h, $v^{\mu\nu}$, b_μ, $f^{\mu\nu\rho}$, and $k_{\mu\nu}$ are some tensors depending at most on the undifferentiated fields Φ^{α_0} from (2). In addition, they display the symmetry/antisymmetry properties

$$s^{\mu\nu\alpha\beta} = -s^{\nu\mu\alpha\beta} = s^{\mu\nu\beta\alpha}, \tag{D.22}$$

$$\tau^{\alpha\beta\mu\nu} = -\tau^{\beta\alpha\mu\nu} = -\tau^{\alpha\beta\nu\mu}, \tag{D.23}$$

$$\tau^{[\alpha\beta\mu]\nu} = 0, \tag{D.24}$$

and $v^{\mu\nu}$, $f^{\mu\nu\rho}$, and $k_{\mu\nu}$ are completely antisymmetric. Because both tensors $s^{\mu\nu\alpha\beta}$ and $\tau^{\alpha\beta\mu\nu}$ are derivative-free, their are related through

$$s^{\mu\nu\alpha\beta} = \tau^{\mu(\alpha\beta)\nu}. \tag{D.25}$$

Using successively properties (D.22)–(D.24) and formula (D.25), it can be shown that $\tau^{\alpha\beta\mu\nu}$ is completely antisymmetric. This last property together with (D.24) leads to

$$\tau^{\alpha\beta\mu\nu} = 0,$$

which replaced in the latter equality from (D.19) produces

$$\frac{\delta \overset{(1)}{\pi}}{\delta t_{\alpha\beta|\mu}} = 0.$$

This means that the entire dependence of $\overset{(1)}{\pi}$ on $t_{\alpha\beta|\mu}$ is trivial (reduces to a full divergence), and therefore $\overset{(1)}{\pi}$ can at most describe self-interactions in the BF sector. Since there is no nontrivial solution to (D.8) that mixes the BF and (2, 1) field sectors, we can safely take

$$\overset{(1)}{\pi} = 0. \tag{D.26}$$

In the end of this section we analyze equation (D.9). Taking one more time into account definitions (45)–(47), it is easy to see that (D.9) implies that the EL derivatives of $\overset{(2)}{\pi}$ are subject to the equations

$$\partial_\mu \frac{\delta \overset{(2)}{\pi}}{\delta t_{\mu(\alpha|\beta)}} = 0, \qquad \partial_\mu \frac{\delta \overset{(2)}{\pi}}{\delta t_{\alpha\beta|\mu}} = 0, \tag{D.27}$$

$$\partial_{[\mu} \frac{\delta \overset{(2)}{\pi}}{\delta H^{\nu]}} = 0, \qquad \partial_\mu \frac{\delta \overset{(2)}{\pi}}{\delta V_\mu} = 0, \qquad \partial_{[\mu} \frac{\delta \overset{(2)}{\pi}}{\delta B^{\nu\rho]}} = 0, \tag{D.28}$$

$$\partial_\mu \frac{\delta \overset{(2)}{\pi}}{\delta \phi_{\mu\nu}} = 0, \qquad \partial_{[\mu} \frac{\delta \overset{(2)}{\pi}}{\delta K^{\nu\rho\lambda]}} = 0. \tag{D.29}$$

Because $\overset{(2)}{\pi}$ (and also its EL derivatives) contains two spacetime derivatives, the solution to both equations from (D.27) is of the type

$$\frac{\delta \overset{(2)}{\pi}}{\delta t_{\mu\nu|\alpha}} = \partial_\rho \partial_\beta \bar{\tau}^{\mu\nu\rho|\alpha\beta}, \tag{D.30}$$

where $\bar{\tau}^{\mu\nu\rho|\alpha\beta}$ depends only on the undifferentiated fields Φ^{α_0} and exhibits the mixed symmetry (3, 2). This means that $\bar{\tau}^{\mu\nu\rho|\alpha\beta}$ is simultaneously antisymmetric in its first three and respectively last two indices and satisfies the identity $\bar{\tau}^{[\mu\nu\rho|\alpha]\beta} = 0$. The solutions to the remaining equations, (D.28) and (D.32), can be represented as

$$\frac{\delta \overset{(2)}{\pi}}{\delta H^\mu} = \partial_\mu \bar{h}, \qquad \frac{\delta \overset{(2)}{\pi}}{\delta V_\mu} = \partial_\nu \bar{v}^{\mu\nu}, \qquad \frac{\delta \overset{(2)}{\pi}}{\delta B^{\mu\nu}} = \partial_{[\mu} \bar{b}_{\nu]}, \tag{D.31}$$

$$\frac{\delta \overset{(2)}{\pi}}{\delta \phi_{\mu\nu}} = \partial_\rho \bar{f}^{\mu\nu\rho}, \qquad \frac{\delta \overset{(2)}{\pi}}{\delta K^{\mu\nu\rho}} = \partial_{[\mu} \bar{k}_{\nu\rho]}, \tag{D.32}$$

where the functions $\bar{v}^{\mu\nu}$, $\bar{f}^{\mu\nu\rho}$, and $\bar{k}_{\mu\nu}$ are completely antisymmetric and contain a single spacetime derivative.

Let N be a derivation in the algebra of the fields $t_{\mu\nu|\alpha}$, H^μ, V_μ, $B^{\mu\nu}$, $\phi_{\mu\nu}$, $K^{\mu\nu\rho}$, and of their derivatives, which counts the powers of these fields and of their derivatives

$$N=\sum_{n\geq 0}\Bigg((\partial_{\mu_1\cdots\mu_n}t_{\mu\nu|\alpha})\frac{\partial}{\partial(\partial_{\mu_1\cdots\mu_n}t_{\mu\nu|\alpha})}+(\partial_{\mu_1\cdots\mu_n}H^{\mu})\frac{\partial}{\partial(\partial_{\mu_1\cdots\mu_n}H^{\mu})}+(\partial_{\mu_1\cdots\mu_n}V_{\mu})\frac{\partial}{\partial(\partial_{\mu_1\cdots\mu_n}V_{\mu})}+(\partial_{\mu_1\cdots\mu_n}B^{\mu\nu})\frac{\partial}{\partial(\partial_{\mu_1\cdots\mu_n}B^{\mu\nu})}+(\partial_{\mu_1\cdots\mu_n}\phi_{\mu\nu})\frac{\partial}{\partial(\partial_{\mu_1\cdots\mu_n}\phi_{\mu\nu})}+(\partial_{\mu_1\cdots\mu_n}K^{\mu\nu\rho})\frac{\partial}{\partial(\partial_{\mu_1\cdots\mu_n}K^{\mu\nu\rho})}\Bigg). \tag{D.33}$$

We emphasize that N does not 'see' either the scalar field φ or its spacetime derivatives. It is easy to check that for every nonintegrated density Ψ we have

$$N\Psi=\frac{\delta\Psi}{\delta t_{\mu\nu|\alpha}}t_{\mu\nu|\alpha}+\frac{\delta\Psi}{\delta H^{\mu}}H^{\mu}+\frac{\delta\Psi}{\delta V_{\mu}}V_{\mu}+\frac{\delta\Psi}{\delta B^{\mu\nu}}B^{\mu\nu}+\frac{\delta\Psi}{\delta\phi_{\mu\nu}}\phi_{\mu\nu}+\frac{\delta\Psi}{\delta K^{\mu\nu\rho}}K^{\mu\nu\rho}+\partial_{\mu}s^{\mu}. \tag{D.34}$$

If $\Psi^{(n)}$ is a homogeneous polynomial of degree n in the fields $t_{\mu\nu|\alpha}$, H^{μ}, V_{μ}, $B^{\mu\nu}$, $\phi_{\mu\nu}$, $K^{\mu\nu\rho}$ and their derivatives (such a polynomial may depend also on φ and its spacetime derivatives, but the homogeneity does not take them into consideration since Ψ is allowed to be a series in φ), then

$$N\Psi^{(n)}=n\Psi^{(n)}.$$

Based on results (D.30)–(D.32), we can write

$$N\overset{(2)}{\pi}=-\frac{1}{3}\bar{\tau}^{\mu\nu\rho|\alpha\beta}R_{\mu\nu\rho|\alpha\beta}-\bar{h}\partial_{\mu}H^{\mu}+\frac{1}{2}\bar{v}^{\mu\nu}\partial_{[\mu}V_{\nu]}+2\bar{b}_{\mu}\partial_{\nu}B^{\mu\nu}-\frac{1}{3}\bar{f}^{\mu\nu\rho}\partial_{[\mu}\phi_{\nu\rho]}-3\bar{k}_{\mu\nu}\partial_{\rho}K^{\mu\nu\rho}+\partial_{\mu}m^{\mu}. \tag{D.35}$$

We decompose $\overset{(2)}{\pi}$ along the degree n as

$$\overset{(2)}{\pi}=\sum_{n\geq 2}\overset{(2)}{\pi}{}^{(n)}, \tag{D.36}$$

where $N\overset{(2)}{\pi}{}^{(n)}=n\overset{(2)}{\pi}{}^{(n)}$ ($n\geq 2$ in (D.36) because $\overset{(2)}{\pi}$, and hence every $\overset{(2)}{\pi}{}^{(n)}$, is assumed to describe cross-interactions between the BF model and the tensor field with the mixed symmetry (2, 1)), and find that

$$N\overset{(2)}{\pi}=\sum_{n\geq 2}n\overset{(2)}{\pi}{}^{(n)}. \tag{D.37}$$

Comparing (D.37) with (D.35), it follows that decomposition (D.36) induces a similar one with respect to each function $\bar{\tau}^{\mu\nu\rho|\alpha\beta}$, $\bar{h}$, $\bar{v}^{\mu\nu}$, $\bar{b}_{\mu}$, $\bar{f}^{\mu\nu\rho}$, and $\bar{k}_{\mu\nu}$

$$\bar{\tau}^{\mu\nu\rho|\alpha\beta}=\sum_{n\geq 2}\bar{\tau}^{\mu\nu\rho|\alpha\beta}_{(n-1)},\qquad \bar{h}=\sum_{n\geq 2}\bar{h}_{(n-1)},\qquad \bar{v}^{\mu\nu}=\sum_{n\geq 2}\bar{v}^{\mu\nu}_{(n-1)}, \tag{D.38}$$

$$\bar{b}^{\mu}=\sum_{n\geq 2}\bar{b}^{\mu}_{(n-1)},\qquad \bar{f}^{\mu\nu\rho}=\sum_{n\geq 2}\bar{f}^{\mu\nu\rho}_{(n-1)},\qquad \bar{k}^{\mu\nu}=\sum_{n\geq 2}\bar{k}^{\mu\nu}_{(n-1)}. \tag{D.39}$$

Inserting (D.38) and (D.39) in (D.35) and comparing the resulting expression with (D.37), we get

$$\overset{(2)}{\pi}{}^{(n)}=-\frac{1}{3n}\bar{\tau}^{\mu\nu\rho|\alpha\beta}_{(n-1)}R_{\mu\nu\rho|\alpha\beta}-\frac{1}{n}\bar{h}_{(n-1)}\partial_{\mu}H^{\mu}+\frac{1}{2n}\bar{v}^{\mu\nu}_{(n-1)}\partial_{[\mu}V_{\nu]}+\frac{2}{n}\bar{b}^{\mu}_{(n-1)}\partial^{\nu}B_{\mu\nu}-\frac{1}{3n}\bar{f}^{\mu\nu\rho}_{(n-1)}\partial_{[\mu}\phi_{\nu\rho]}-\frac{3}{n}\bar{k}^{\mu\nu}_{(n-1)}\partial^{\rho}K_{\mu\nu\rho}+\partial_{\mu}m^{\mu}_{(n)}. \tag{D.40}$$

Replacing the last result, (D.40), into (D.36), we further obtain

$$\overset{(2)}{\pi}=-\frac{1}{3}\hat{\tau}^{\mu\nu\rho|\alpha\beta}R_{\mu\nu\rho|\alpha\beta}-\hat{h}\partial_{\mu}H^{\mu}+\frac{1}{2}\hat{v}^{\mu\nu}\partial_{[\mu}V_{\nu]}+2\hat{b}_{\mu}\partial_{\nu}B^{\mu\nu}-\frac{1}{3}\hat{f}^{\mu\nu\rho}\partial_{[\mu}\phi_{\nu\rho]}-3\hat{k}_{\mu\nu}\partial_{\rho}K^{\mu\nu\rho}+\partial_{\mu}\hat{m}^{\mu}, \tag{D.41}$$

where

$$\hat{\tau}^{\mu\nu\rho|\alpha\beta}=\sum_{n\geq 2}\frac{1}{n}\bar{\tau}^{\mu\nu\rho|\alpha\beta}_{(n-1)},\qquad \hat{h}=\sum_{n\geq 2}\frac{1}{n}\bar{h}_{(n-1)},\qquad \hat{v}^{\mu\nu}=\sum_{n\geq 2}\frac{1}{n}\bar{v}^{\mu\nu}_{(n-1)}, \tag{D.42}$$

$$\hat{b}^{\mu}=\sum_{n\geq 2}\frac{1}{n}\bar{b}^{\mu}_{(n-1)},\qquad \hat{f}^{\mu\nu\rho}=\sum_{n\geq 2}\frac{1}{n}\bar{f}^{\mu\nu\rho}_{(n-1)},\qquad \hat{k}^{\mu\nu}=\sum_{n\geq 2}\frac{1}{n}\bar{k}^{\mu\nu}_{(n-1)}. \tag{D.43}$$

So far, we showed that the solution to (D.9) can be put in the form (D.41). By means of definitions (36)–(37), we can bring (D.41) to the expression

$$\overset{(2)}{\pi} = -\frac{1}{3}\hat{\tau}^{\mu\nu\rho|\alpha\beta}R_{\mu\nu\rho|\alpha\beta} + \partial_\mu \hat{m}^\mu + \delta\big(-\varphi^*\hat{h} - B^*_{\mu\nu}\hat{v}^{\mu\nu} - 2V^*_\mu\hat{b}^\mu + K^*_{\mu\nu\rho}\hat{f}^{\mu\nu\rho} - 3\phi^{*\mu\nu}\hat{k}_{\mu\nu}\big). \tag{D.44}$$

The δ-exact modulo d terms in the right-hand side of (D.44) produce purely trivial interactions, which can be eliminated via field redefinitions. This is due to the isomorphism $H^i(s|d) \simeq H^i(\gamma|d, H_0(\delta))$ in all positive values of the ghost number and respectively of the pure ghost number [42], which at $i = 0$ allows one to state that any solution of (D.9) that is δ-exact modulo d is in fact a trivial cocycle from $H^0(s|d)$. In conclusion, the only nontrivial solution to (D.9) can be written as

$$\overset{(2)}{\pi} = -\frac{1}{3}\hat{\tau}^{\mu\nu\rho|\alpha\beta}R_{\mu\nu\rho|\alpha\beta}, \tag{D.45}$$

where $\hat{\tau}^{\mu\nu\rho|\alpha\beta}$ displays the mixed symmetry $(3,2)$, is derivative-free, and is required to depend at least on one field from the BF sector. But $R_{\mu\nu\rho|\alpha\beta}$ already contains two spacetime derivatives, so such a $\overset{(2)}{\pi}$ disagrees with the hypothesis on the differential order of the interacting field equations (see also the discussion following formula (D.4)), which means that we must set

$$\overset{(2)}{\pi} = 0. \tag{D.46}$$

Substituting results (D.15), (D.26), and (D.46) into decomposition (D.6), we obtain

$$\bar{a}_0''^{\text{int}} = 0, \tag{D.47}$$

which combined with (D.5) proves that indeed there is no nontrivial solution to the 'homogeneous' equation (119) that complies with all the working hypotheses

$$\bar{a}_0^{\text{int}} = 0. \tag{D.48}$$

Appendix E: Notations from Sect. 6

In this Appendix we list the concrete form of the various notations made in Sect. 6.

The polynomials denoted by $\bar{X}_p^{(i)}$ that enter $\bar{\Delta}^{\text{int}}$ given in (137) read

$$\bar{X}_0^{(1)} = 6S^*\eta C + 12t^{*\mu}(V_\mu C + \eta C_\mu) + 6\big(2B^*_{\mu\nu}C + V_{[\mu}C_{\nu]} - \phi_{\mu\nu}\eta\big)F^{\mu\nu} - 2\big(2\eta^*_{\mu\nu\rho}C + 2B^*_{[\mu\nu}C_{\rho]} - 3K^*_{\mu\nu\rho}\eta - \phi_{[\mu\nu}V_{\rho]}\big)\tilde{D}_{\lambda\sigma}\varepsilon^{\mu\nu\rho\lambda\sigma}, \tag{E.1}$$

$$\bar{X}_1^{(1)} = \big[\big(-2C^*_{\mu\nu\rho}\eta - 2C^*_{[\mu\nu}V_{\rho]} - 4H^*_{[\mu}B^*_{\nu\rho]}\big)C + \big(-2H^*_{[\mu}V_\nu C_{\rho]} + 2H^*_{[\mu}\phi_{\nu\rho]}\eta\big) + 2C^*_{[\mu\nu}C_{\rho]}\eta\big]\tilde{D}_{\lambda\sigma}\varepsilon^{\mu\nu\rho\lambda\sigma} - 12H^*_\mu t^{*\mu}\eta C + 6\big(H^*_{[\mu}V_{\nu]}C + 2H^*_\mu\eta C_\nu + C^*_{\mu\nu}\eta C\big)F^{\mu\nu}, \tag{E.2}$$

$$\bar{X}_2^{(1)} = \big[\big(-2H^*_{[\mu}C^*_{\nu\rho]}\eta - 2H^*_{[\mu}H^*_\nu V_{\rho]}\big)C + 2H^*_{[\mu}H^*_\nu C_{\rho]}\eta\big]\tilde{D}_{\lambda\sigma}\varepsilon^{\mu\nu\rho\lambda\sigma} + 6H^*_\mu H^*_\nu\eta C F^{\mu\nu}, \tag{E.3}$$

$$\bar{X}_3^{(1)} = 4H^*_\mu H^*_\nu H^*_\rho\eta C D^{\mu\nu\rho}, \tag{E.4}$$

$$\bar{X}_0^{(2)} = -12\cdot 5!\big(S^*\eta + 2t^{*\mu}V_\mu + 2B^*_{\mu\nu}F^{\mu\nu}\big)\tilde{\mathcal{G}} - 4\cdot 5!\eta^*_{\mu\nu\rho}\tilde{D}_{\lambda\sigma}\mathcal{G}^{\mu\nu\rho\lambda\sigma} + 4!\cdot 4!t^{*\mu}\eta\tilde{\mathcal{G}}_\mu - 4!\cdot 4!B^*_{\mu\nu}\tilde{D}_{\rho\lambda}\mathcal{G}^{\mu\nu\rho\lambda} + 6\cdot 4!\big(\phi^{*\mu\nu}\eta - K^{\mu\nu\rho}V_\rho\big)\tilde{D}_{\mu\nu} - 3\cdot 4!\big(\tilde{K}_{\mu\nu}\eta - 4V_{[\mu}\tilde{\mathcal{G}}_{\nu]}\big)F^{\mu\nu}, \tag{E.5}$$

$$\bar{X}_1^{(2)} = -4\cdot 5!\big(C^*_{\mu\nu\rho}\eta + C^*_{[\mu\nu}V_{\rho]} + 2H^*_{[\mu}B^*_{\nu\rho]}\big)\tilde{D}_{\lambda\sigma}\mathcal{G}^{\mu\nu\rho\lambda\sigma} - 12\cdot 5!\big(C^*_{\mu\nu}F^{\mu\nu}\eta - 2H^*_\mu t^{*\mu}\eta + H^*_{[\mu}V_{\nu]}F^{\mu\nu}\big)\tilde{\mathcal{G}} - 12\cdot 4!H^*_{[\mu}\tilde{\mathcal{G}}_{\nu]}\eta F^{\mu\nu} - 12\cdot 4!\big(C^*_{\mu\nu}\eta + H^*_{[\mu}V_{\nu]}\big)\tilde{D}_{\rho\lambda}\mathcal{G}^{\mu\nu\rho\lambda} - 6\cdot 4!H^*_\mu K^{\mu\nu\rho}\eta\tilde{D}_{\nu\rho}, \tag{E.6}$$

$$\bar{X}_2^{(2)} = -4\cdot 5!\big(H^*_{[\mu}C^*_{\nu\rho]}\eta + H^*_{[\mu}H^*_\nu V_{\rho]}\big)\tilde{D}_{\lambda\sigma}\mathcal{G}^{\mu\nu\rho\lambda\sigma} - 12\cdot 4!H^*_\mu H^*_\nu\eta\tilde{D}_{\rho\lambda}\mathcal{G}^{\mu\nu\rho\lambda} - 12\cdot 5!H^*_\mu H^*_\nu\eta F^{\mu\nu}\tilde{\mathcal{G}}, \tag{E.7}$$

$$\bar{X}_3^{(2)} = -4\cdot 5!H^*_\mu H^*_\nu H^*_\rho\eta\tilde{D}_{\lambda\sigma}\mathcal{G}^{\mu\nu\rho\lambda\sigma}, \tag{E.8}$$

$$\bar{X}_0^{(3)} = -6\cdot 5!S^*\tilde{\eta} + 12\cdot 4!t^*_\mu\tilde{\eta}^\mu + 4!B^{\mu\nu}\tilde{D}_{\mu\nu} - 36\tilde{\eta}_{\mu\nu}F^{\mu\nu}, \tag{E.9}$$

$$\bar{X}_1^{(3)} = 2\cdot 5!C^*_{\mu\nu\rho}\tilde{D}_{\lambda\sigma}\eta^{\mu\nu\rho\lambda\sigma} + 6\cdot 5!\big(2H^*_\mu t^{*\mu} - C^*_{\mu\nu}F^{\mu\nu}\big)\tilde{\eta} + 6\cdot 4!C^*_{\mu\nu}\tilde{D}_{\rho\lambda}\eta^{\mu\nu\rho\lambda} + 3\cdot 4!H^*_\mu\tilde{D}_{\nu\rho}\eta^{\mu\nu\rho} + 6\cdot 4!H^*_{[\mu}\tilde{\eta}_{\nu]}F^{\mu\nu}, \tag{E.10}$$

$$\bar{X}_2^{(3)} = 2\cdot 5!H^*_{[\mu}C^*_{\nu\rho]}\tilde{D}_{\lambda\sigma}\eta^{\mu\nu\rho\lambda\sigma} + 6\cdot 4!H^*_\mu H^*_\nu\big(\tilde{D}_{\rho\lambda}\eta^{\mu\nu\rho\lambda} - 5F^{\mu\nu}\tilde{\eta}\big), \tag{E.11}$$

$$\bar{X}_3^{(3)} = 2 \cdot 5! H^*_\mu H^*_\nu H^*_\rho \tilde{D}_{\lambda\sigma} \eta^{\mu\nu\rho\lambda\sigma}. \tag{E.12}$$

The functions appearing in (146) and denoted by $U_p^{(i)}$ are of the form

$$U_0^{(1)} = -9\left(2k_1\phi^{\mu\nu} - \frac{k_2}{10}\tilde{K}^{\mu\nu}\right) \times \left(2B^*_{\mu\nu}C + V_{[\mu}C_{\nu]} - \phi_{\mu\nu}\eta\right), \tag{E.13}$$

$$U_1^{(1)} = -9\left(k_1\phi^{\mu\nu} - \frac{k_2}{20}\tilde{K}^{\mu\nu}\right) \times \left[C^*_{\mu\nu}\eta C - H^*_\mu(V_\nu C + \eta C_\nu)\right], \tag{E.14}$$

$$U_2^{(1)} = -9\left(k_1\phi^{\mu\nu} - \frac{k_2}{20}\tilde{K}^{\mu\nu}\right) H^*_\mu H^*_\nu \eta C, \tag{E.15}$$

$$U_0^{(2)} = 108\left(k_1\phi^{\mu\nu} - \frac{k_2}{20}\tilde{K}^{\mu\nu}\right) \times \left(40B^*_{\mu\nu}\tilde{\mathcal{G}} + \eta\tilde{K}_{\mu\nu} - 8V_\mu\tilde{\mathcal{G}}_\nu\right), \tag{E.16}$$

$$U_1^{(2)} = 18\varepsilon^{\alpha\beta\gamma\delta\varepsilon}\left(C^*_{\mu\nu}\eta + H^*_{[\mu}V_{\nu]}\right) \times \left(k_1\phi^{\mu\nu} - \frac{k_2}{20}\tilde{K}^{\mu\nu}\right)\mathcal{G}_{\alpha\beta\gamma\delta\varepsilon} - 36\varepsilon_{\rho\alpha\beta\gamma\delta}H^*_\mu\left(k_1\phi^{\mu\rho} - \frac{k_2}{20}\tilde{K}^{\mu\rho}\right)\eta\mathcal{G}^{\alpha\beta\gamma\delta}, \tag{E.17}$$

$$U_2^{(2)} = 18\varepsilon_{\alpha\beta\gamma\delta\varepsilon}\left(k_1\phi^{\mu\nu} - \frac{k_2}{20}\tilde{K}^{\mu\nu}\right) \times H^*_\mu H^*_\nu \eta\mathcal{G}^{\alpha\beta\gamma\delta\varepsilon}, \tag{E.18}$$

$$U_0^{(3)} = 9\varepsilon_{\nu\rho\alpha\beta\gamma}\left(k_1\phi^{\nu\rho} - \frac{k_2}{20}\tilde{K}^{\nu\rho}\right)\eta^{\alpha\beta\gamma}, \tag{E.19}$$

$$U_1^{(3)} = \frac{9}{4}\varepsilon_{\alpha\beta\gamma\delta\varepsilon}C^*_{\mu\nu}\left(k_1\phi^{\mu\nu} - \frac{k_2}{20}\tilde{K}^{\mu\nu}\right)\eta^{\alpha\beta\gamma\delta\varepsilon} - 18\varepsilon_{\rho\beta\gamma\delta\varepsilon}H^*_\mu\left(k_1\phi^{\mu\rho} - \frac{k_2}{20}\tilde{K}^{\mu\rho}\right)\eta^{\beta\gamma\delta\varepsilon}, \tag{E.20}$$

$$U_2^{(3)} = 9\varepsilon_{\alpha\beta\gamma\delta\varepsilon}H^*_\mu H^*_\nu\left(k_1\phi^{\mu\nu} - \frac{k_2}{20}\tilde{K}^{\mu\nu}\right)\eta^{\alpha\beta\gamma\delta\varepsilon}. \tag{E.21}$$

Appendix F: Deformed gauge structure

If we denote by $\Omega_1^{\alpha_1}$ and $\Omega_2^{\alpha_1}$ two independent sets of gauge parameters,

$$\Omega_1^{\alpha_1} \equiv \left(\epsilon^{(1)\mu\nu}, \epsilon^{(1)}, \epsilon^{(1)\mu\nu\rho}, \xi_\mu^{(1)}, \xi^{(1)\mu\nu\rho\lambda}, \theta_{\mu\nu}^{(1)}, \chi_{\mu\nu}^{(1)}\right), \tag{F.1}$$

$$\Omega_2^{\alpha_1} \equiv \left(\epsilon^{(2)\mu\nu}, \epsilon^{(2)}, \epsilon^{(2)\mu\nu\rho}, \xi_\mu^{(2)}, \xi^{(2)\mu\nu\rho\lambda}, \theta_{\mu\nu}^{(2)}, \chi_{\mu\nu}^{(2)}\right), \tag{F.2}$$

then the concrete form of the commutators among the deformed gauge transformations of the fields associated with (F.1) and (F.2) (and generically written as in (162)) read

$$[\bar{\delta}_{\Omega_1}, \bar{\delta}_{\Omega_2}]\varphi = 0, \tag{F.3}$$

$$[\bar{\delta}_{\Omega_1}, \bar{\delta}_{\Omega_2}]H^\mu = \bar{\delta}_\Omega H^\mu - 2\frac{\delta S^{\mathrm{L}}}{\delta H^\nu}\frac{d\epsilon^{\mu\nu}}{d\varphi} - 3\frac{\delta S^{\mathrm{L}}}{\delta B^{\nu\rho}}\frac{d\epsilon^{\mu\nu\rho}}{d\varphi} + 2\frac{\delta S^{\mathrm{L}}}{\delta\phi_{\mu\nu}}\frac{d\xi_\nu}{d\varphi} - 4\frac{\delta S^{\mathrm{L}}}{\delta K^{\nu\rho\lambda}}\frac{d\xi^{\mu\nu\rho\lambda}}{d\varphi}, \tag{F.4}$$

$$[\bar{\delta}_{\Omega_1}, \bar{\delta}_{\Omega_2}]V_\mu = \bar{\delta}_\Omega V_\mu, \tag{F.5}$$

$$[\bar{\delta}_{\Omega_1}, \bar{\delta}_{\Omega_2}]B^{\mu\nu} = \bar{\delta}_\Omega B^{\mu\nu} + 3\frac{\delta S^{\mathrm{L}}}{\delta H^\rho}\frac{d\epsilon^{\mu\nu\rho}}{d\varphi}, \tag{F.6}$$

$$[\bar{\delta}_{\Omega_1}, \bar{\delta}_{\Omega_2}]\phi_{\mu\nu} = \bar{\delta}_\Omega\phi_{\mu\nu} - \frac{\delta S^{\mathrm{L}}}{\delta H^{[\mu}}\frac{d\xi_{\nu]}}{d\varphi}, \tag{F.7}$$

$$[\bar{\delta}_{\Omega_1}, \bar{\delta}_{\Omega_2}]K^{\mu\nu\rho} = \bar{\delta}_\Omega K^{\mu\nu\rho} - 4\frac{\delta S^{\mathrm{L}}}{\delta H^\lambda}\frac{d\xi^{\mu\nu\rho\lambda}}{d\varphi}, \tag{F.8}$$

$$[\bar{\delta}_{\Omega_1}, \bar{\delta}_{\Omega_2}]t_{\mu\nu|\alpha} = 0. \tag{F.9}$$

The gauge parameters from the right-hand side of the above formulas are defined through

$$\Omega^{\alpha_1} = \left(\epsilon^{\mu\nu}, \epsilon = 0, \epsilon^{\mu\nu\rho}, \xi_\mu, \xi^{\mu\nu\rho\lambda}, \theta_{\mu\nu} = 0, \chi_{\mu\nu} = 0\right), \tag{F.10}$$

where

$$\begin{aligned}\epsilon^{\mu\nu} = \lambda\Bigg\{&-\frac{dW_1}{d\varphi}\left(\epsilon^{(1)}\epsilon^{(2)\mu\nu} - \epsilon^{(2)}\epsilon^{(1)\mu\nu}\right) \\ &+ 6\frac{dW_3}{d\varphi}\Bigg[\phi_{\rho\lambda}\left(\epsilon^{(1)}\xi^{(2)\mu\nu\rho\lambda} - \epsilon^{(2)}\xi^{(1)\mu\nu\rho\lambda}\right) \\ &+ \frac{1}{2}K^{\mu\nu\rho}\left(\epsilon^{(1)}\xi_\rho^{(2)} - \epsilon^{(2)}\xi_\rho^{(1)}\right) \\ &- 2V_\rho\left(\xi_\lambda^{(1)}\xi^{(2)\mu\nu\rho\lambda} - \xi_\lambda^{(2)}\xi^{(1)\mu\nu\rho\lambda}\right)\Bigg] \\ &- 3\frac{dW_2}{d\varphi}\left(\xi_\rho^{(1)}\epsilon^{(2)\mu\nu\rho} - \xi_\rho^{(2)}\epsilon^{(1)\mu\nu\rho}\right) \\ &+ 3\frac{dW_6}{d\varphi}\varepsilon_{\rho\alpha\beta\gamma\delta}\left(\epsilon^{(1)\mu\nu\rho}\xi^{(2)\alpha\beta\gamma\delta} - \epsilon^{(2)\mu\nu\rho}\xi^{(1)\alpha\beta\gamma\delta}\right) \\ &+ 6\frac{dW_4}{d\varphi}\Bigg[\varepsilon_{\rho\alpha\beta\gamma\delta}K^{\mu\nu\rho}\left(\epsilon^{(1)}\xi^{(2)\alpha\beta\gamma\delta} - \epsilon^{(2)}\xi^{(1)\alpha\beta\gamma\delta}\right) \\ &+ \frac{1}{6}\varepsilon^{\mu\nu\rho\lambda\sigma}\varepsilon_{\lambda\alpha\beta\gamma\delta}\varepsilon_{\sigma\alpha'\beta'\gamma'\delta'}V_\rho\xi^{(1)\alpha\beta\gamma\delta}\xi^{(2)\alpha'\beta'\gamma'\delta'}\Bigg] \\ &- \frac{1}{2}\varepsilon^{\mu\nu\rho\lambda\sigma}\frac{dW_5}{d\varphi}\Bigg[\phi_{\rho\lambda}\left(\epsilon^{(1)}\xi_\sigma^{(2)} - \epsilon^{(2)}\xi_\sigma^{(1)}\right) \\ &- 2V_\rho\xi_\lambda^{(1)}\xi_\sigma^{(2)}\Bigg]\Bigg\},\end{aligned} \tag{F.11}$$

$$\epsilon^{\mu\nu\rho} = -8\lambda\Bigg[W_3\left(\xi_\lambda^{(1)}\xi^{(2)\mu\nu\rho\lambda} - \xi_\lambda^{(2)}\xi^{(1)\mu\nu\rho\lambda}\right) - \frac{1}{12}\varepsilon^{\mu\nu\rho\lambda\sigma}\Big(W_4\varepsilon_{\lambda\alpha\beta\gamma\delta}\varepsilon_{\sigma\alpha'\beta'\gamma'\delta'}\xi^{(1)\alpha\beta\gamma\delta}\xi^{(2)\alpha'\beta'\gamma'\delta'}$$

$$+ W_5\xi_\lambda^{(1)}\xi_\sigma^{(2)}\Big)\Bigg], \tag{F.12}$$

$$\xi_\mu = -3\lambda\big[W_3\big(\epsilon^{(1)}\xi_\mu^{(2)} - \epsilon^{(2)}\xi_\mu^{(1)}\big) + 2W_4\varepsilon_{\mu\nu\rho\lambda\sigma}\big(\epsilon^{(1)}\xi^{(2)\nu\rho\lambda\sigma} - \epsilon^{(2)}\xi^{(1)\nu\rho\lambda\sigma}\big)\big], \tag{F.13}$$

$$\xi^{\mu\nu\rho\lambda} = 3\lambda\Bigg[W_3\big(\epsilon^{(1)}\xi^{(2)\mu\nu\rho\lambda} - \epsilon^{(2)}\xi^{(1)\mu\nu\rho\lambda}\big) - \frac{1}{12}W_4\varepsilon^{\mu\nu\rho\lambda\sigma}\big(\epsilon^{(1)}\xi_\sigma^{(2)} - \epsilon^{(2)}\xi_\sigma^{(1)}\big)\Bigg]. \tag{F.14}$$

In addition, we made the notations

$$\theta^{(i)} = \sigma_{\alpha\beta}\theta^{(i)\alpha\beta}, \quad i = \overline{1,2}. \tag{F.15}$$

Related to the first-order reducibility, the transformations (163) are given by

$$\epsilon^{\mu\nu}(\bar{\Omega}) = -3D_\rho\bar{\epsilon}^{\mu\nu\rho} - \lambda\frac{dW_2}{d\varphi}\big(B^{\mu\nu}\bar{\xi} - 6\phi_{\rho\lambda}\bar{\epsilon}^{\mu\nu\rho\lambda}\big) + 3\lambda\frac{dW_3}{d\varphi}V_\rho\big(K^{\mu\nu\rho}\bar{\xi} - 10\phi_{\lambda\sigma}\bar{\xi}^{\mu\nu\rho\lambda\sigma}\big) - 6\lambda\varepsilon_{\alpha\beta\gamma\delta\varepsilon}\frac{dW_4}{d\varphi}K^{\mu\nu\rho}V_\rho\bar{\xi}^{\alpha\beta\gamma\delta\varepsilon} - \frac{\lambda}{2}\varepsilon^{\mu\nu\rho\lambda\sigma}\frac{dW_5}{d\varphi}V_\rho\phi_{\lambda\sigma}\bar{\xi} + \lambda\frac{dW_6}{d\varphi}\big(\varepsilon_{\alpha\beta\gamma\delta\varepsilon}B^{\mu\nu}\bar{\xi}^{\alpha\beta\gamma\delta\varepsilon} + 3K^{\mu\nu\rho}\varepsilon_{\rho\alpha\beta\gamma\delta}\bar{\epsilon}^{\alpha\beta\gamma\delta}\big), \tag{F.16}$$

$$\epsilon(\bar{\Omega}) = 2\lambda\big(W_2\bar{\xi} - \varepsilon_{\alpha\beta\gamma\delta\varepsilon}W_6\bar{\xi}^{\alpha\beta\gamma\delta\varepsilon}\big), \tag{F.17}$$

$$\epsilon^{\mu\nu\rho}(\bar{\Omega}) = 4\partial_\lambda\bar{\epsilon}^{\mu\nu\rho\lambda} + 2\lambda W_1\bar{\epsilon}^{\mu\nu\rho} - 20\lambda W_3\phi_{\lambda\sigma}\bar{\xi}^{\mu\nu\rho\lambda\sigma} + 2\lambda K^{\mu\nu\rho}\big(W_3\bar{\xi} - 2\varepsilon_{\alpha\beta\gamma\delta\varepsilon}W_4\bar{\xi}^{\alpha\beta\gamma\delta\varepsilon}\big) - \frac{\lambda}{3}\varepsilon^{\mu\nu\rho\lambda\sigma}W_5\phi_{\lambda\sigma}\bar{\xi}, \tag{F.18}$$

$$\xi_\mu(\bar{\Omega}) = D_\mu^{(-)}\bar{\xi} + 6\lambda\varepsilon_{\alpha\beta\gamma\delta\varepsilon}W_4V_\mu\bar{\xi}^{\alpha\beta\gamma\delta\varepsilon} - 3\lambda\varepsilon_{\mu\nu\rho\lambda\sigma}W_6\bar{\epsilon}^{\nu\rho\lambda\sigma}, \tag{F.19}$$

$$\xi^{\mu\nu\rho\lambda}(\bar{\Omega}) = -5D_\sigma^{(+)}\bar{\xi}^{\mu\nu\rho\lambda\sigma} + 3\lambda W_2\bar{\epsilon}^{\mu\nu\rho\lambda} - \frac{\lambda}{4}\varepsilon^{\mu\nu\rho\lambda\sigma}W_5V_\sigma\bar{\xi}, \tag{F.20}$$

$$\theta_{\mu\nu}(\bar{\Omega}) = 3\partial_{(\mu}\bar{\theta}_{\nu)} + \lambda\sigma_{\mu\nu}\left(k_1\bar{\xi} + \frac{k_2}{5!}\varepsilon_{\alpha\beta\gamma\delta\varepsilon}\bar{\xi}^{\alpha\beta\gamma\delta\varepsilon}\right), \tag{F.21}$$

$$\chi_{\mu\nu}(\bar{\Omega}) = \partial_{[\mu}\bar{\theta}_{\nu]}, \tag{F.22}$$

while the first-order reducibility relations (164) read

$$\bar{\delta}_{\Omega(\bar{\Omega})}\varphi = 0, \tag{F.23}$$

$$\bar{\delta}_{\Omega(\bar{\Omega})}H^\mu = \lambda\frac{\delta S^{\mathrm{L}}}{\delta H^\nu}\Bigg\{6V_\rho\Bigg[\frac{d^2W_1}{d\varphi^2}\bar{\epsilon}^{\mu\nu\rho} - \frac{d^2W_3}{d\varphi^2}\big(10\phi_{\lambda\sigma}\bar{\xi}^{\mu\nu\rho\lambda\sigma} - K^{\mu\nu\rho}\bar{\xi}\big) - 2\varepsilon_{\alpha\beta\gamma\delta\varepsilon}\frac{d^2W_4}{d\varphi^2}K^{\mu\nu\rho}\bar{\xi}^{\alpha\beta\gamma\delta\varepsilon} - \frac{1}{6}\varepsilon^{\mu\nu\rho\lambda\sigma}\frac{d^2W_5}{d\varphi^2}\phi_{\lambda\sigma}\bar{\xi}\Bigg] + 2\frac{d^2W_6}{d\varphi^2}\big(3\varepsilon_{\rho\alpha\beta\gamma\delta}K^{\mu\nu\rho}\bar{\epsilon}^{\alpha\beta\gamma\delta} + \varepsilon_{\alpha\beta\gamma\delta\varepsilon}B^{\mu\nu}\bar{\xi}^{\alpha\beta\gamma\delta\varepsilon}\big) + 2\frac{d^2W_2}{d\varphi^2}\big(6\phi_{\rho\lambda}\bar{\epsilon}^{\mu\nu\rho\lambda} - B^{\mu\nu}\bar{\xi}\big)\Bigg\} + 6\lambda\frac{\delta S^{\mathrm{L}}}{\delta B^{\nu\rho}}\Bigg[\frac{dW_1}{d\varphi}\bar{\epsilon}^{\mu\nu\rho} - \frac{dW_3}{d\varphi}\big(10\phi_{\lambda\sigma}\bar{\xi}^{\mu\nu\rho\lambda\sigma} - K^{\mu\nu\rho}\bar{\xi}\big) - 2\varepsilon_{\alpha\beta\gamma\delta\varepsilon}\frac{dW_4}{d\varphi}K^{\mu\nu\rho}\bar{\xi}^{\alpha\beta\gamma\delta\varepsilon} - \frac{1}{6}\varepsilon^{\mu\nu\rho\lambda\sigma}\frac{dW_5}{d\varphi}\phi_{\lambda\sigma}\bar{\xi}\Bigg] - 2\lambda\frac{\delta S^{\mathrm{L}}}{\delta V_\mu}\left(\frac{dW_2}{d\varphi}\bar{\xi} - \varepsilon_{\alpha\beta\gamma\delta\varepsilon}\frac{dW_6}{d\varphi}\bar{\xi}^{\alpha\beta\gamma\delta\varepsilon}\right) + \lambda\frac{\delta S^{\mathrm{L}}}{\delta K^{\nu\rho\lambda}}\Bigg[-V_\sigma\left(60\frac{dW_3}{d\varphi}\bar{\xi}^{\mu\nu\rho\lambda\sigma} + \varepsilon^{\mu\nu\rho\lambda\sigma}\frac{dW_5}{d\varphi}\bar{\xi}\right) + 12\frac{dW_2}{d\varphi}\bar{\epsilon}^{\mu\nu\rho\lambda}\Bigg] + 6\lambda\frac{\delta S^{\mathrm{L}}}{\delta\phi_{\mu\nu}}\Bigg[\varepsilon_{\nu\alpha\beta\gamma\delta}\frac{dW_6}{d\varphi}\bar{\epsilon}^{\alpha\beta\gamma\delta} + V_\nu\left(\frac{dW_3}{d\varphi}\bar{\xi} - 2\varepsilon_{\alpha\beta\gamma\delta\varepsilon}\frac{dW_4}{d\varphi}\bar{\xi}^{\alpha\beta\gamma\delta\varepsilon}\right)\Bigg], \tag{F.24}$$

$$\bar{\delta}_{\Omega(\bar{\Omega})}V_\mu = 2\lambda\frac{\delta S^{\mathrm{L}}}{\delta H^\mu}\left(\frac{dW_2}{d\varphi}\bar{\xi} - \varepsilon_{\alpha\beta\gamma\delta\varepsilon}\frac{dW_6}{d\varphi}\bar{\xi}^{\alpha\beta\gamma\delta\varepsilon}\right), \tag{F.25}$$

$$\bar{\delta}_{\Omega(\bar{\Omega})}B^{\mu\nu} = 6\lambda\frac{\delta S^{\mathrm{L}}}{\delta H^\rho}\Bigg[-\frac{dW_1}{d\varphi}\bar{\epsilon}^{\mu\nu\rho} + 10\frac{dW_3}{d\varphi}\phi_{\lambda\sigma}\bar{\xi}^{\mu\nu\rho\lambda\sigma} - K^{\mu\nu\rho}\left(\frac{dW_3}{d\varphi}\bar{\xi} - 2\frac{dW_4}{d\varphi}\varepsilon_{\alpha\beta\gamma\delta\varepsilon}\bar{\xi}^{\alpha\beta\gamma\delta\varepsilon}\right)$$

$$+\frac{1}{6}\varepsilon^{\mu\nu\rho\lambda\sigma}\frac{dW_5}{d\varphi}\phi_{\lambda\sigma}\bar{\xi}\Bigg]$$
$$+\lambda\frac{\delta S^{\mathrm{L}}}{\delta K^{\rho\lambda\sigma}}\left(60W_3\bar{\xi}^{\mu\nu\rho\lambda\sigma}+\varepsilon^{\mu\nu\rho\lambda\sigma}W_5\bar{\xi}\right)$$
$$+6\lambda\frac{\delta S^{\mathrm{L}}}{\delta\phi_{\mu\nu}}\left(W_3\bar{\xi}-2\varepsilon_{\alpha\beta\gamma\delta\varepsilon}W_4\bar{\xi}^{\alpha\beta\gamma\delta\varepsilon}\right), \quad \text{(F.26)}$$

$$\bar{\delta}_{\Omega(\bar{\Omega})}\phi_{\mu\nu}=-3\lambda\frac{\delta S^{\mathrm{L}}}{\delta H^{[\mu}}V_{\nu]}\left(\frac{dW_3}{d\varphi}\bar{\xi}-2\varepsilon_{\alpha\beta\gamma\delta\varepsilon}\frac{dW_4}{d\varphi}\bar{\xi}^{\alpha\beta\gamma\delta\varepsilon}\right)$$
$$-6\lambda\frac{\delta S^{\mathrm{L}}}{\delta B^{\mu\nu}}\left(W_3\bar{\xi}-2\varepsilon_{\alpha\beta\gamma\delta\varepsilon}W_4\bar{\xi}^{\alpha\beta\gamma\delta\varepsilon}\right)$$
$$-3\lambda\frac{dW_6}{d\varphi}\frac{\delta S^{\mathrm{L}}}{\delta H^{[\mu}}\varepsilon_{\nu]\alpha\beta\gamma\delta}\bar{\epsilon}^{\alpha\beta\gamma\delta}, \quad \text{(F.27)}$$

$$\bar{\delta}_{\Omega(\bar{\Omega})}K^{\mu\nu\rho}=\lambda\frac{\delta S^{\mathrm{L}}}{\delta H^{\lambda}}\Bigg[-V_\sigma\left(60\frac{dW_3}{d\varphi}\bar{\xi}^{\mu\nu\rho\lambda\sigma}+\varepsilon^{\mu\nu\rho\lambda\sigma}\frac{dW_5}{d\varphi}\bar{\xi}\right)+12\frac{dW_2}{d\varphi}\bar{\epsilon}^{\mu\nu\rho\lambda}\Bigg]$$
$$-\lambda\frac{\delta S^{\mathrm{L}}}{\delta B^{\lambda\sigma}}\left(60W_3\bar{\xi}^{\mu\nu\rho\lambda\sigma}+\varepsilon^{\mu\nu\rho\lambda\sigma}W_5\bar{\xi}\right), \quad \text{(F.28)}$$

$$\bar{\delta}_{\Omega(\bar{\Omega})}t_{\mu\nu|\alpha}=0. \quad \text{(F.29)}$$

Regarding the second-order reducibility, the transformations (165) take the concrete form

$$\bar{\epsilon}^{\mu\nu\rho}(\check{\Omega})=4D_\lambda\check{\epsilon}^{\mu\nu\rho\lambda}-\lambda\left(10\frac{dW_2}{d\varphi}\phi_{\lambda\sigma}\check{\epsilon}^{\mu\nu\rho\lambda\sigma}+\varepsilon_{\alpha\beta\gamma\delta\varepsilon}\frac{dW_6}{d\varphi}K^{\mu\nu\rho}\check{\epsilon}^{\alpha\beta\gamma\delta\varepsilon}\right), \quad \text{(F.30)}$$
$$\bar{\epsilon}^{\mu\nu\rho\lambda}(\check{\Omega})=-5\partial_\sigma\check{\epsilon}^{\mu\nu\rho\lambda\sigma}-2\lambda W_1\check{\epsilon}^{\mu\nu\rho\lambda},$$
$$\bar{\xi}(\check{\Omega})=-3\lambda\varepsilon_{\alpha\beta\gamma\delta\varepsilon}W_6\check{\epsilon}^{\alpha\beta\gamma\delta\varepsilon}, \quad \text{(F.31)}$$
$$\bar{\xi}^{\mu\nu\rho\lambda\sigma}(\check{\Omega})=-3\lambda W_2\check{\epsilon}^{\mu\nu\rho\lambda\sigma}, \qquad \bar{\theta}_\mu(\check{\Omega})=0, \quad \text{(F.32)}$$

such that the second-order reducibility relations (166) become

$$\epsilon^{\mu\nu}\left(\bar{\Omega}(\check{\Omega})\right)=3\lambda\frac{\delta S^{\mathrm{L}}}{\delta H^{\rho}}\left(4\frac{d^2W_1}{d\varphi^2}V_\lambda\check{\epsilon}^{\mu\nu\rho\lambda}+10\frac{d^2W_2}{d\varphi^2}\phi_{\lambda\sigma}\check{\epsilon}^{\mu\nu\rho\lambda\sigma}+\varepsilon_{\alpha\beta\gamma\delta\varepsilon}\frac{d^2W_6}{d\varphi^2}K^{\mu\nu\rho}\check{\epsilon}^{\alpha\beta\gamma\delta\varepsilon}\right)$$
$$+12\lambda\frac{dW_1}{d\varphi}\frac{\delta S^{\mathrm{L}}}{\delta B^{\rho\lambda}}\check{\epsilon}^{\mu\nu\rho\lambda}+30\lambda\frac{dW_2}{d\varphi}\frac{\delta S^{\mathrm{L}}}{\delta K^{\rho\lambda\sigma}}\check{\epsilon}^{\mu\nu\rho\lambda\sigma}$$
$$-3\lambda\varepsilon_{\alpha\beta\gamma\delta\varepsilon}\frac{dW_6}{d\varphi}\frac{\delta S^{\mathrm{L}}}{\delta\phi_{\mu\nu}}\check{\epsilon}^{\alpha\beta\gamma\delta\varepsilon}, \quad \text{(F.33)}$$

$$\epsilon\left(\bar{\Omega}(\check{\Omega})\right)=0, \quad \text{(F.34)}$$

$$\epsilon^{\mu\nu\rho}\left(\bar{\Omega}(\check{\Omega})\right)=-8\lambda\frac{dW_1}{d\varphi}\frac{\delta S^{\mathrm{L}}}{\delta H^{\lambda}}\check{\epsilon}^{\mu\nu\rho\lambda}, \quad \text{(F.35)}$$

$$\xi_\mu\left(\bar{\Omega}(\check{\Omega})\right)=-3\lambda\varepsilon_{\alpha\beta\gamma\delta\varepsilon}\frac{dW_6}{d\varphi}\frac{\delta S^{\mathrm{L}}}{\delta H^{\mu}}\check{\epsilon}^{\alpha\beta\gamma\delta\varepsilon}, \quad \text{(F.36)}$$

$$\xi^{\mu\nu\rho\lambda}\left(\bar{\Omega}(\check{\Omega})\right)=15\lambda\frac{dW_2}{d\varphi}\frac{\delta S^{\mathrm{L}}}{\delta H^{\sigma}}\check{\epsilon}^{\mu\nu\rho\lambda\sigma}, \quad \text{(F.37)}$$

$$\theta_{\mu\nu}\left(\bar{\Omega}(\check{\Omega})\right)=0, \qquad \chi_{\mu\nu}\left(\bar{\Omega}(\check{\Omega})\right)=0. \quad \text{(F.38)}$$

Finally, we investigate the third-order reducibility, for which the transformations (167) can be written as

$$\check{\epsilon}^{\mu\nu\rho\lambda}(\hat{\Omega})=-5D_\sigma\hat{\epsilon}^{\mu\nu\rho\lambda\sigma}, \quad \text{(F.39)}$$
$$\check{\epsilon}^{\mu\nu\rho\lambda\sigma}(\hat{\Omega})=2\lambda W_1\hat{\epsilon}^{\mu\nu\rho\lambda\sigma}, \quad \text{(F.40)}$$

while that for the third-order reducibility relations (168) are listed below:

$$\bar{\epsilon}^{\mu\nu\rho}\left(\check{\Omega}(\hat{\Omega})\right)=20\lambda\left(\frac{\delta S^{\mathrm{L}}}{\delta H^{\lambda}}\frac{d^2W_1}{d\varphi^2}V_\sigma+\frac{\delta S^{\mathrm{L}}}{\delta B^{\lambda\sigma}}\frac{dW_1}{d\varphi}\right)\hat{\epsilon}^{\mu\nu\rho\lambda\sigma}, \quad \text{(F.41)}$$

$$\bar{\epsilon}^{\mu\nu\rho\lambda}\left(\check{\Omega}(\hat{\Omega})\right)=-10\lambda\frac{\delta S^{\mathrm{L}}}{\delta H^{\sigma}}\frac{dW_1}{d\varphi}\hat{\epsilon}^{\mu\nu\rho\lambda\sigma} \quad \text{(F.42)}$$

$$\bar{\xi}\left(\check{\Omega}(\hat{\Omega})\right)=0,$$
$$\bar{\xi}^{\mu\nu\rho\lambda\sigma}\left(\check{\Omega}(\hat{\Omega})\right)=0, \quad \text{(F.43)}$$
$$\bar{\theta}_\mu\left(\check{\Omega}(\hat{\Omega})\right)=0.$$

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